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MONOTONICITY OF THE DECAY OF UNSTABLE PARTICLES CORRESPONDING TO AN n-ORDER POLE

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It was pointed out in a recent paper [1] that the energy distribution of unstable particles $\omega(E)$ may possibly correspond not to a first-order pole, as is usually assumed, but to a pole of order n:

$$\omega_n(E) = C_n[(E - E_0)^2 + (\Gamma^2/4)]^n \tag{1}$$

In order for these values of $\omega_n(E)$ actually to correspond to the energy distribution of the unstable particle, it is necessary, of course [2], that the decay of such a particle be monotonic 1), i.e.,

$$[dL_n(t)]/dt \leq 0, \qquad t \in [0, \infty) \tag{2}$$

where $L_n(t) = |p_n(t)|^2$ is the law of decay of the unstable particle with energy distribution (1). In [2] we presented a complete description of all the energy distributions $\omega(E)$ of the unstable particles, i.e., all the $\omega(E)$ for which the monotonicity condition is satisfied. We can prove, by verifying the necessary and sufficient conditions indicated in [2], that the $\omega_n(E)$ belong to the class of energy distributions of physical systems.

It is simpler, however, to prove this directly. It was shown in [1] that 2)

$$p_n(t) = \exp\left\{-\frac{i}{\hbar}E_0 t - \frac{\Gamma t}{\hbar}\right\}\sum_{\ell=0}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-1}\frac{(n+\ell-1)!(n-1)}{(n-\ell-1)!\ell!(2n-2)!} \tag{3}$$

and consequently

$$\frac{dL_n(t)}{dt} = -\frac{\Gamma}{\hbar}\exp\left\{-\frac{\Gamma}{\hbar}\right\}\sum_{k=0}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-k-1}\frac{(n+k-1)!(n-1)!}{(n-k-1)!k!(2n-2)!}$$
$$\times\left\{\sum_{\ell=0}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-1}\frac{(n+\ell-1)!(n-1)!}{(n-\ell-1)!\ell!(2n-2)!} - 2\sum_{\ell=0}^{n-2}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-2}\frac{(n-\ell-1)(n+\ell-1)!(n-1)!}{(n-\ell-1)!\ell!(2n-2)!}\right\} \tag{4}$$

Inasmuch as it is obvious that

$$\sum_{k=0}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-k-1}\frac{(n+k-1)!(n-1)!}{(n-k-1)!k!(2n-2)!}\geq 0, \qquad t \in [0, \infty) \tag{5}$$

to prove (2) it is sufficient to show that

$$\sum_{\ell=0}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-1}\frac{(n+\ell-1)!(n-1)!}{(n-\ell-1)!\ell!(2n-2)!}\geq 2\sum_{\ell=0}^{n-2}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-2}\frac{(n-\ell-1)(n+\ell-1)(n-1)!}{(n-\ell-1)!\ell!(2n-2)!} \tag{6}$$

For the proof, we make in the right side of (6) the substitution $\ell' = \ell + 1$ and then

$$\sum_{\ell=0}^{n-2}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-2}\frac{(n-\ell-1)(n+\ell-1)!(n-1)!}{(n-\ell-1)!\ell!(2n-2)!} = \sum_{\ell'=1}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell'-1}\frac{\ell'}{n+\ell'-1}\;\frac{(n+\ell'-1)!(n-1)!}{(n-\ell'-1)!\ell'!(2n-2)!} \tag{7}$$

Thus, (6) is equivalent to

$$\left(\frac{\Gamma t}{\hbar}\right)^{n-1}\frac{(n-1)!}{(2n-2)!}+\sum_{\ell=1}^{n-1}\left(\frac{\Gamma t}{\hbar}\right)^{n-\ell-1}\frac{(n+\ell-1)!(n-1)!}{(n-\ell-1)!\ell!(2n-2)!}\left[1-\frac{2\ell}{n+\ell-1}\right]\geq 0 \tag{8}$$

But

$$1-\frac{2\ell}{n+\ell-1}=\frac{n-\ell-1}{n+\ell-1}\geq 0 \tag{9}$$

since $\ell = 1, 2, \ldots, n-1$. The inequality (8) is therefore true, thus proving (6) and consequently also (2).

The energy distribution corresponding to a pole of order n is thus actually acceptable, for arbitrary n, as the energy distribution of an unstable particle, inasmuch as the decay law is monotonic.

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1) This condition is not metnioned in [1].

2) Only the principal term of the decay law is written out in (3).