HEATING OF IONS BY NONLINEAR ABSORPTION BY TRANSVERSE WAVES IN A PLASMA

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In linear theory there is no collisionless damping of transverse waves in a nonmagnetized plasma, and the absorption of the waves is connected only with the presence of electron-ion collisions. Practically the entire absorbed energy goes in this case to heating of the electrons. On the other hand, the energy of the ions increases only as a result of their collision with hot electrons. In this paper we wish to call attention to a possibility of direct heating of ions by nonlinear absorption of transverse waves as a result of induced scattering of transverse waves by the plasma ions. Taking these processes into account, the equation for the ionic distribution function $F(\underset{\sim}{v},t)$ can be written in the form [1]

$$\partial F/\partial t = (\partial/\partial v_i)D_{ij}(\partial F/\partial v_j) \qquad (i,j = 1,2,3)$$

$$D_{ij} = (1/2M^2)\int d\underset{\sim}{k}d\underset{\sim}{k}_1 \frac{I(\underset{\sim}{k})I(\underset{\sim}{k}_1)}{\omega\omega_1}(k_i - k_{1i})(k_j - k_{1j})W^{t,t}(\underset{\sim}{k}, \underset{\sim}{k}_1) \tag{1}$$

Here 1)

$$W^{t,t}(\underset{\sim}{k},\underset{\sim}{k}_1) = \frac{\pi Z^2}{4}\,\frac{\omega_0^4}{n^2}\,\frac{\delta(\Omega - \underset{\sim}{q}\cdot\underset{\sim}{v})}{\omega\omega_1}\left|\frac{\epsilon_e^{l}(\Omega,\underset{\sim}{q}) - 1}{\epsilon^{l}(\Omega,\underset{\sim}{q})}\right|^2(1 + \cos^2\widehat{\underset{\sim}{k}\,\underset{\sim}{k}_1}) \tag{2}$$

$$(\underset{\sim}{q} = \underset{\sim}{k} - \underset{\sim}{k}_1, \qquad \Omega = \omega - \omega_1)$$

is the probability of scattering of an ion with velocity $\underset{\sim}{v}$, accompanied by absorption of a photon with wave vector $\underset{\sim}{k}_1$ and frequency $\omega_1 = (\omega_0^2 + k_1^2c^2)^{1/2}$, and by emission of a photon with wave vector $\underset{\sim}{k}$ and frequency $\omega = (\omega_0^2\ \omega\ k^2c^2)^{1/2}$, M and Z are the mass and charge number of the ion, c is the velocity of light, ω_0 is the plasma frequency, n the electron density, $\epsilon^{l} = \epsilon_e^{l} + \epsilon_i^{l} - 1$ is the longitudinal dielectric constant of the electron-ion plasma, and finally I(k) is the spectral radiation density of the plasma, normalized so that $\int I(\underset{\sim}{k})dk = U$, where U is the average energy density of the transverse waves. Strictly speaking, Equ. (1) must be solved simultaneously with the equation for the spectral density $I(\underset{\sim}{k})$. However, bearing in mind that under laboratory conditions the plasma dimensions are, as shown by simple calculations, as a rule much smaller than the characteristic length of the nonlinear absorption, we can neglect in (1) the change of $I(\underset{\sim}{k})$ due to absorption, and $I(\underset{\sim}{k})$ can be regarded as a specified function defined by the power of the external sources. Without embarking on a complete investigation of (1), we confine ourselves here to an examination of two particular cases which, on the one hand, are sufficiently simple from the mathematical point of view, and on the other are of practical interest.

1. We assume that two beams of transverse waves with identical spectral distribution propagate in a plasma head-on along the z axis, so that

$$I(\underset{\sim}{k}) = (d\omega/dk)I_0(\omega)\delta(k_x)\delta(k_y); \quad \omega = (\omega_0^2 + k^2c^2)^{1/2} \tag{3}$$

where $I_0(\omega)$ is the spectral energy density in one of the beams per unit frequency interval ω, so that $\int I_0(\omega) = U$. In this case Eq. (1) takes the form

$$\partial F/\partial t = (\partial/\partial v_z)D_{\parallel}(v_z)(\partial F/\partial v_z); \quad k^2(\omega) = [(\omega^2 - \omega_0^2)/c^2]$$
$$D_{\parallel}(v_z) = \pi\left[\frac{ZT_i}{T_i + ZT_e}\right]^2 \frac{\omega_0^4}{n^2M^2}\int_{\omega_0}^{\infty} d\omega\frac{k^2\omega}{\omega^4} I_0(\omega)\left[I_0(\omega + 2kv_z) + I_0(\omega - 2kv_z)\right] \tag{4}$$

where T_e and T_i are the electron and ion temperatures. It follows from (4) that the nonlinear absorption increases only the z-component of the velocity, parallel to the propagation direction of the colliding waves.

We denote by $\Delta\omega \ll \omega_0$ the characteristic width of the spectrum and assume that $I(\omega)$ differs from zero only inside the interval $[\bar{\omega}, \bar{\omega} + \Delta\omega]$.[2] Then, as follows from the expression for the coefficient $D_{\parallel}(v_z)$, this coefficient differs from zero only when $v_z < v_{max} = \Delta\omega/2k(\bar{\omega})$. In other words, the effective increase of the ion energy continues only so long as the average thermal velocity of the ion v_{T_i} does not exceed the value v_{max}, which thus determines in fact the maximum ion energy ($\epsilon_{max} = Mv_{max}^2/2$) attainable with this method of heating. Assuming that the function $I_0(\omega) \approx U/\Delta\omega$ varies little in the interval $\Delta\omega$, we obtain

$$D_{\parallel}(v_z) = 2\pi\left(\frac{ZT_i}{T_i + ZT_e}\right)^2 \frac{\omega_0^4}{n^2M^2}\frac{U^2k^2(\bar{\omega})}{\bar{\omega}^4\Delta\omega}\begin{cases}[1 - (v_z/v_{max})] & v_z \leq v_{max}\\ 0 & v_z \geq v_{max}\end{cases} \tag{5}$$

If the initial ion temperature is sufficiently low, so that $v_{T_i} \ll v_{max}$, we can neglect the dependence of the diffusion coefficient on v_z, and obtain for the rate of change of the average ion kinetic energy [3]

$$\partial\epsilon/\partial t = (\partial/\partial t)(1/Zn)\int(Mv^2/2)Fd\underset{\sim}{v} = \epsilon_{max}/\tau; \quad t \lesssim \tau \tag{6}$$

where the characteristic time is

$$\tau = \frac{\epsilon_{max}}{MD_{\parallel}}\frac{1}{16\pi}\left(\frac{T_i + ZT_e}{ZT_i}\right)^2 \frac{\bar{\omega}^4}{\omega_0^4}\frac{\Delta\omega}{\bar{\omega}^2 - \omega_0^2}\left(\frac{\Delta\omega}{\bar{\omega}}\right)^2\left(\frac{nMc^2}{U}\right)^2 \tag{7}$$

2. We now consider a second case, when the radiation intensity does not depend on the angles, i.e.,

$$I(\underset{\sim}{k}) = (d\omega/dk)[I_0(\omega)/4\pi k^2(\omega)] \tag{8}$$

The heating process will then obviously have an isotropic character and Eq. (1) assumes the form

$$\partial F/\partial t = (1/v^2)(\partial/\partial v)v^2D(v)(\partial F/\partial v),$$
$$D(v) = \frac{\pi}{8}\left(\frac{ZT_i}{T_i + ZT_e}\right)^2 \frac{\omega_0^4}{n^2M^2}\int_{\omega_0}^{\infty}\frac{d\omega I_0(\omega)}{v^3\omega^4 k(\omega)}\int_{\omega}^{\omega+2kv} d\omega_1(\omega - \omega_1)^2\left[1 + \left(\frac{\omega - \omega_1}{kv} + 1\right)^2\right]I_0(\omega_1) \tag{9}$$

It follows therefore that although in this case the diffusion coefficient d(v) does not vanish for any value of the velocity v, it is practically independent of v when $v \ll v_{max}$, and begins to decrease like v^{-3} when $v \gg v_{max}$. The latter, as in one-dimensional case, causes a most effective heating of the ions only so long as the ion energy $\epsilon(t)$ does not exceed ϵ_{max}, after which it slows down sharply. If $I_0(\omega) \approx U/\Delta\omega$ varies little in the interval $\Delta\omega$, then

$$D(v) = \frac{7\pi}{15}\left(\frac{ZT_i}{T_i+ZT_e}\right)^2 \frac{\omega_0^4}{n^2M^2} \frac{U^2k^2(\omega)}{\bar{\omega}^4\Delta^\omega} \begin{cases} 1 - \frac{11}{14}\frac{v}{v_{max}} & \text{if } v \leq v_{max} \\ \frac{5}{14}\left(\frac{v_{max}}{v}\right)^3\left[1 - \frac{6}{5}\frac{v_{max}}{v} + \frac{4}{5}\left(\frac{v_{max}}{v}\right)^2\right] & \text{if } v \geq v_{max} \end{cases} \qquad (10)$$

Consequently, at sufficiently low ion temperatures, when $\epsilon(0) \ll \epsilon_{max}$, the quantity v/v_{max} in (10) can be neglected and the rate of energy increase is determined by relation (6), except that the numerical coefficient 1/16 in expression (7) for the characteristic time must be replaced by 5/56.

By way of an example, we estimate the heating efficiency by assuming that the electrons and ions of a hydrogen plasma with density $n \leq 3 \times 10^{11}$ cm^{-3} have initially identical temperatures, $\bar{\omega} = 10^{11}$ sec^{-1}, $U = 10^2$ erg/cm^3, and $\Delta\omega/\omega = 10^{-3}$ (corresponding to a high-frequency field intensity on the order of 10 kV and to $\epsilon_{max} \approx 120$ eV). We then find that $\tau \simeq 10^{-5}$ sec, i.e., the ion energy increases to ~ 120 eV within ~ 10 μsec.

Thus, the mechanism of nonlinear absorption considered above can serve as one method for directly heating plasma ions. It must be emphasized, however, that, owing to the presence of the factor $[ZT_i/(T_i + ZT_e)]^2$ in the expression for the diffusion coefficient, the efficiency of this method depends essentially on the ratio of the electron and ion temperatures, and decreases sharply for a non-isothermal plasma with $T_e \gg T_i$.

[1] L. Kovrizhnykh, JETP 48, 1114 (1965), Soviet Phys. JETP 21, 744 (1965).

1) We confine ourselves to the case of a non-magnetized plasma, when the external magnetic field is equal to zero.

2) Simple estimates show that for sufficiently narrow spectra, when $\Delta\omega/\omega \ll [mT_i^2/MT_e^2]^{1/4}$ (m - electron mass), it is possible in practice to neglect the electron energy change connected with the nonlinear absorption.

3) When $t > \tau$ the rate of increase of the ion energy slows down sharply.