We consider further the following Jacobi identity

$$J(E_\alpha, P_\rho, E'_\gamma) = a^{\beta}_{\alpha\rho}[E_\beta, E'_\gamma] + b^{j}_{\alpha\rho}[H_j, E'_\gamma] + c^{\tau}_{\alpha\rho}[P_\rho E'_\gamma] + d^{l}_{\alpha\rho}[H'_l, E'_\gamma] + f^{\nu}_{\alpha\rho}[E'_\nu, E'_\gamma] = 0 \tag{11}$$

Taking (3) and (10) into account, it follows from (11) that

$$d^{l}_{\alpha\rho} = 0 \tag{12}$$

To prove that $a_{\alpha\rho} = 0$, it is sufficient to use the Jacobi identity

$$J(P_\rho, P_\sigma, E_\alpha) \equiv 0 \tag{13}$$

and the properties of the constants $\lambda^{\tau}_{\rho\sigma}$ (see [1]).

2. $$[E'_\gamma, H_i] = D^{\nu}_{\gamma i}E'_\nu, \quad [P_\rho, H'_l] = C^{m}_{\rho l}H'_m, \quad [H'_l, E_\alpha] = 0 \tag{14}$$

3. If there exists in G at least one generator of H'_l which commutes with the generators of P and S, then in this case, too, $[E_\alpha, P_\rho] = 0$.

To prove these statements it is necessary to use in lieu of (11) the identity

$$J(E_\alpha, P_\rho, H'_l) \equiv 0 \tag{15}$$

In conclusion we note that if condition (6) is not satisfied for all i, then, as shown in [2], we can construct a nontrivial union G, the generators of which will be only P_ρ, H_i, and E_α.

[1] Coester, Hamermesh, and McGlinn, Phys. Rev. 135, B451 (1964).

[2] Ottson, Kihlberg, and Nilsson, Preprint, Gothenburg, 1964.

[3] L. Michel, Phys. Rev. 137, B405 (1965).

[4] Gursey, Pais, and Radicati, Phys. Rev. Lett. 13, 239 (1964).

[5] Kadyshevskii, Muradyan, Tavkhelidze, and Todorov, JINR Preprint D-1929, 1964.

POSSIBLE SUPERCONDUCTIVITY MECHANISM IN CRYSTALLINE FILMS

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New superconductivity mechanisms have been proposed in many recent papers [1,2]. We consider below one possibility of establishment of a superconducting state, due to the presence of different groups of electrons in a crystalline film. The interaction between the electrons of these groups leads, if certain conditions are satisfied (see below), to Cooper pairing. For a bulky crystal, the non-phonon superconductivity mechanism, due to interband interaction, was considered by Geilikman [2], who established a general criterion, which is

of significance also to the case considered below, wherein the interelectron interaction leads to effective attraction.

Different groups of electronic states (subbands) arise in the film because of the finite character of the transverse motion of the electrons (holes), and in each n-th group the state is given by the longitudinal projection of the quasimomentum. In addition, in semiconducting films (see [3,4] concerning superconductivity in semiconductors) the degeneracy of the edge of the band, frequently encountered in bulky samples, is lifted because of the decreasy in symmetry [5], leading to the formation of two or more two-dimensional bands, with the edges of the bands shifting relative to one another, and with the electrons differing in their effective masses and wave functions.

For a quantitative calculation let us consider the following model, which corresponds to the possible band structure of a semiconducting film. There are two groups of electronic states. For concreteness we shall assume that the minimum electron energy ϵ_{m1} lies lower than ϵ_{m2}. We denote by μ_1 and μ_2 the chemical potential measured from ϵ_{m1} and ϵ_{m2} respectively. In our case μ depends on the temperature.

The Coulomb interaction of the electrons of the first and second groups leads to an effective interaction between the electrons of the first group. Let us write an equation for the proper-energy part Δ_{11}:

$$\Delta_{11}(p,\omega_n) = [T/(2\pi)^3]\sum_{\omega_{n'}}\int dp\Gamma_{11}(p',\omega_{n'};\ p',\omega_{n'};\ p,\omega_n;\ p,\omega_n)F^{+}_{11}(p',\omega_{n'}) \qquad (1)$$

where Γ_{11} is the complete four-pole, $F^{+}_{11} = -i\langle T(\Psi^{+}_1(x)\Psi^{+}_1(x'))\rangle$ [6], and $\omega_n = (2n+1)\pi T$.

A general expression for Γ_{11} obtained by Geilikman by a diagram technique [2] is:

$$\Gamma_{11} = (V_{11} + \Pi_{22}R)/S \qquad (2)$$

where $R = V_{12}^2 - V_{11}V_{22}$; V_{11}, V_{22}, and V_{12} are the scattering matrix elements whereby the electrons remain in their groups. Π_{22} is the polarization operator, with $\Pi_{22} < 0$, $S = 1 - V_{11}\Pi_{11} - V_{22}\Pi_{22} - \Pi_{11}\Pi_{22}R$. A distinguishing feature of the quantity Γ_{11} in our case is the temperature dependence of Π_{22} (see below), which causes the constant describing the effective interelectron interaction to be a function of the temperature, in contrast with the usual case.

It follows from (2) that attraction occurs when two conditions are satisfied: first, if $R > 0$ and, second, if $\Pi_{22}R$ is sufficiently large compared with the renormalized Coulomb repulsion. When the sample becomes thicker, the difference in the matrix elements vanishes ($R \to 0$), leading to a vanishing of the effect, so that the latter is possible only in thin films.

Calculation of Π_{22} leads to the expression

$$\Pi_{22}(\kappa, T) = -2\exp(-|\mu_2|/T)\sum_{p}\frac{\exp(-\epsilon_{p}/T) - \exp(-\epsilon_{p+\kappa}/T)}{i\omega - \epsilon_p + \epsilon_{p+\kappa}} \qquad (3)$$

where κ is the momentum transfer. It has been assumed that Boltzmann statistics hold for the electrons of the upper group. For small $\kappa^2 \ll m_2T$, Π_{22} is a constant and equal to

$m_2V \exp(-|\mu|/T)\hbar^2L$, while for $\kappa^2 \gg m_2T$ we have $\Pi_{22} \sim \kappa^{-2}$, i.e., it approaches zero quite rapidly. We can therefore assume that the attraction is constant in an energy region of the order of T and is equal to zero outside it.

Carrying out further the summation in (1), we obtain

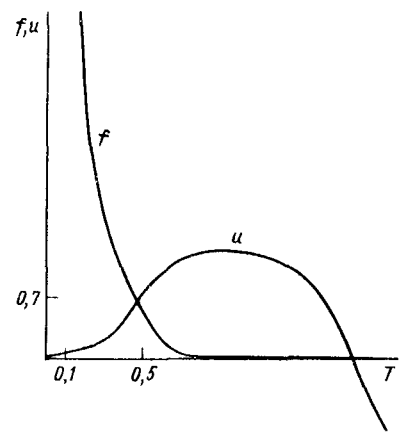
$$Q/g(T)N = \int \frac{d\xi \tanh[(\xi^2 + \Delta^2)^{1/2}/2T]}{(\xi^2 + \Delta^2)^{1/2}} \qquad (4)$$

where $\xi = \epsilon - \mu_1(T)$, N is the density of the states in the film per unit volume, equal to $N = 2\pi m_1/\hbar^2L$; L is the thickness of the film, $g = \Gamma \cdot 2V = g_{attr} - \tilde{g}$, where $\tilde{g}$ is the renormalized Coulomb repulsion; $\omega_c \gg T$, with ω_c the plasma frequency.

The transition temperature is determined from (4) by putting $\Delta = 0$. In the nondegenerate case ($\tanh(\xi/2T_k) = 1$) we obtain

$$\hbar^4L^2\kappa_0^4 \exp(|\mu_2|/T_k)/m_1m_2e^4 = \ln|[T_k - \mu(T_k)]/\mu(T_k)| \qquad (5)$$

In a semiconducting film we have $\mu = -E\alpha/2 + T \ln(N_d\pi\hbar^2/mT)/2$.



Equation (5) can be solved graphically. The curves f and u of the Figure correspond to the left and right sides of (4) with the parameters indicated in the text: for $T < 1°$ the curves were calculated from formula (5).

We see that the effect can occur only at a finite temperature, and if the superconductivity criterion is satisfied and the effect exists in the temperature interval $T_{ku} \geq T_{k\ell}$. An estimate for $L \simeq 5 \times 10^{-6}$, $\kappa \simeq 5 \times 10^5$, $E_d = 5°$, and $N_d = 5 \times 10^{18}$ yields, for example, $T_{k\ell} \simeq 0.5°$ for the lower limit.

In the degenerate case, so long as $\mu_2 < 0$, the lower critical temperature remains different from zero, as can be readily seen from (4). In the case of a metallic film, the mechanism considered is likewise possible in principle. A distinguishing feature in this case is the population of many planes in the Brillouin zone. The inequality $\mu_n > 0$ is satisfied for any n-th group of electrons. It would be interesting from this point of view to investigate experimentally thin films of metals which are not superconductors in bulk. A detailed theoretical consideration of this problem will be presented elsewhere.

Superconducting semiconductors have been recently observed. In connection with the results obtained in this paper, it would be of interest to set up an experiment with superconducting films. The mechanism considered above can lead, first, to the absence of an isotopic effect and, in conjunction with the usual phonon mechanism, to its dependence on the thickness of the film; second, it can disclose the existence of a temperature $T_{k\ell}$, below which there is no superconductivity. The presence of $T_{k\ell}$, as shown above, is due to the temperature dependence of the number of carriers and the effective interelectron interaction.

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[1] W. A. Little, Phys. Rev. 134, A1416 (1964); V. L. Ginzburg, JETP 47, 2318 (1964), Soviet Phys. JETP 20, 1549 (1965); S. V. Vosnovskii and M. S. Svirskii, JETP 47, 1354 (1964), Soviet Phys. JETP 20, 914 (1965).

[2] B. T. Geilikman, JETP 48, 1194 (1965), Soviet Phys. JETP 21, 796 (1965).

[3] Gurevich, Larkin, and Firsov, FTT 4, 185 (1962), Soviet Phys. Solid State 4, 131 (1962); M. L. Cohen, Phys. Rev. 134, A511 (1964).

[4] B. A. Tavger and V. Ya. Demikhovskii, JETP 48, 748 (1965), Soviet Phys. JETP 21, 494 (1965).

[5] B. A. Tavger, JETP 48, 185 (1965), Soviet Phys. JETP 21, 125 (1965).

[6] L. P. Gor'kov, JETP 34, 735 (1958), Soviet Phys. JETP 7, 505 (1958).

[7] B. A. Tavger and V. Ya. Demikhovskii, FTT 5, 644 (1963), Soviet Phys. Solid State 5, 469 (1963).

[8] Scholey, Hosler, and Cohen, Phys. Rev. Lett. 12, 1174 (1964).

FILLING OF ELECTRON SHELLS OF COMPRESSED ATOMS IN THE STATISTICAL MODEL

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A statistical analysis of the electrons of the atom makes it possible to show sufficiently simly that at high pressures the electrons with given quantum number l can first appear in elements whose atomic numbers are smaller than the atomic numbers of the periodic system.

In the simple Thomas-Fermi (TF) model, it is most convenient to solve the problem with the aid of the Lenz-Jensen variational method [1], since the existing solution of the TF equation for the compressed atom [2] does not make it possible to vary its radius continuously. On the other hand, in a model in which quantum corrections are taken into account [3], the general solution of the problem is possible only with the aid of a variational method. By virtue of the normalization condition for the compressed atom, the electron-density distribution function can be chosen only in the zeroth approximation.

In the TF model for a compressed atom we have

$$\rho = (Z\lambda^3/16\pi)[e^{-\Lambda}/\Lambda^3(1 - \gamma)] \qquad (1)$$

where $\Lambda = (\lambda R)^{1/2}$, λ is a variational parameter determined from the condition that the total energy of the atomic numbers be minimal, $\gamma = (1 + \Lambda + \Lambda^2/2)e^{-\Lambda}$ is a correction term, and Z is the atomic number of the element.

Starting from the simplified Sommerfeld condition [4], we can show that for a compressed atom the first appearance of s-, p-, d-, and f-electrons will be determined by the formula