a perfectly realistic requirement for the conditions of this experiment.

The weakening of the field in the center of the discharge by the skin effect can lead to a decrease in the average energy of the electrons in this region, accompanied by attenuation of the radiation intensity on the axis while the overall intensity increases. The maximum of the intensity should shift in this case to the peripheral zone. To check on this assumption, we made additional measurements with a narrower radiation beam at a pressure (0.53 mm Hg) corresponding to the most pronounced maximum. We obtained the ratios of the radiation intensities (δ) of the annular and axial zones of the plasma. This ratio is shown in Fig. 2 as a function of the discharge power. At 45 watts, corresponding to the start of the decrease of the curves on Fig. 1 at microwave power, δ begins to decrease rapidly, showing that the maximum of the intensity shifts to the peripheral region and confirming the foregoing assumption.

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SELF FOCUSING OF WAVE BEAMS IN NONLINEAR MEDIA

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References [1,2] contain calculations of cylindrical self-maintaining waveguide channels (2- and 3-dimensional) in an isotropic nonlinear dielectric with wave number $k_n = k[\epsilon(E_0^2)]^{1/2}$ ($k = (\omega/c)n_0$ is the wave number as $E_0 \rightarrow 0$, and E_0 is the field amplitude). We discuss below certain features of paraxial wave beams in such a medium in the case of weak nonlinearity of ϵ: 1)

$$\epsilon = 1 + \epsilon' E_0^2; \quad \epsilon' E_0^2 \ll 1; \quad \epsilon' > 0 \tag{1}$$

Under the assumptions customarily made in quasi-optics concerning the character of the wave beam [2], which make it possible to disregard the polarization effect and the longitudinal diffusion in the ray amplitude for E_0 and the slowly varying part φ of the phase beam $E = E_0 \exp(ik\tilde{z} - i\varphi + i\omega t)$, we obtain the following equations (in dimensionless coordinates $k\tilde{x}$, $k\tilde{y}$, $k\tilde{z}$)

$$\partial E_0^2/\partial z = -\mathrm{div}_\perp(E_0^2 \nabla_\perp \varphi); \quad 2\varphi_z' + (\nabla_\perp \varphi)^2 = \epsilon_{\mathrm{eff}} - 1 \tag{2}$$

$$\epsilon_{\mathrm{eff}} = \epsilon(E_0^2) + (\Delta_\perp E_0/E_0) \tag{3}$$

which are equivalent to the equation for the transverse diffusion of the ray amplitude of the field [3]. A consequence of (2) is the following equation for paraxial rays

$$\ddot{\vec{r}}_\perp = \frac{1}{2}\nabla_\perp \epsilon_{\mathrm{eff}} \tag{4}$$

according to which the first term in (3) determines the refraction of the rays in an inhomogeneous dielectric, and the second determines their diffraction bending.

For a specified beam profile, we can asses from the character of the right side of (4) the degree of focusing (or defocusing) as the beam enters the nonlinear medium. Thus, when a beam with a transverse profile similar to that of a stationary (cylindrical) beam [1,2] is incident on a plane boundary of a nonlinear medium, it becomes focused over the entire cross section when the total power is $P > P_{st}$ and defocused when $P < P_{st}$, where P_{st} is the power in the nonlinear medium of a stationary beam having the same dimensions as the incident beam. We note that in the two-dimensional case the power P_{st} is inversely proportional to the width $\underline{a}$, while in the three-dimensional case it is independent of the width of the beam (when $ka \gg 1$).

When $\epsilon' E_0^2 \gg (k\Lambda_\perp)^{-2}$, where $\Lambda_\perp$ is the characteristic scale of variation of the field in the cross section of the beam, we can neglect the term $\Delta_\perp E_0/E_0$ in (2) (geometric-optics approximation). One of the solutions of practical interest in this is a spherical wave with variable center of curvature

$$\varphi = [K(z)r^2/2] + \varphi_0(z); \quad \epsilon' E_0^2 = 2\varphi_0' + r^2(K' + K^2) \tag{5}$$

an analysis of which discloses many singularities in the beam structure at the maximum (minimum) of the field. It can be shown that

$$K(z) = D(z - z_0)/1 + D(z - z_0)^2 \tag{6}$$

The constants D and z_0 are determined by the structure of the beam (5) on the boundary $z = 0$ of the nonlinear medium. Beams described by expressions (5) and (6) are qualitatively different when $D > 0$ (defocusing profile $\epsilon(z = 0)$) and when $D < 0$ (focusing profile $\epsilon(z = 0)$). They are shown schematically in Fig. 1. The points $z_{1,2} = z_0 \pm |D|^{-1/2}$ of Fig. 1b are focal points, and solution (5) is not valid in their vicinity. Therefore the beam sections separated by these points in Fig. 1b must be considered independently. The passage of the beam through the focus calls for a special analysis, which is beyond the limits of the employed approximation.

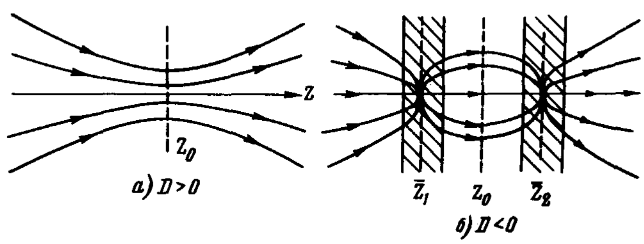

Fig. 1

The most significant aspects of the phenomenon of self focusing of beams, with account taken of the diffraction term $\Delta_\perp E_0/E_0$ in Eq. (2), can be explained by using the following approximation of $\epsilon(E_0^2)$:

$$\epsilon - 1 = \epsilon' E_0^2 = \epsilon' E_M^2 f \quad \epsilon' E_M^2(1 + B \ln f) \equiv \epsilon' E_M^2 \tilde{f} \tag{7}$$

where $f = E_0^2/E_M^2$, with E_M the value of the field at the maximum of the beam. When $B = (s - 1) \times (\ln s)^{-1}$, the approximating function $\tilde{f}$ coincides with f at two points, $f = 1$ and $f = s$. The functions differ most in the interval (1, s) at the point $f = B$, by an amount $1 - B(1 - \ln B)$. When $s = 0.3$ ($B = 0.6$) the difference does not exceed 0.1. Choosing s equal to this value, we can use the approximation (7) for the central part of the bounded beam.

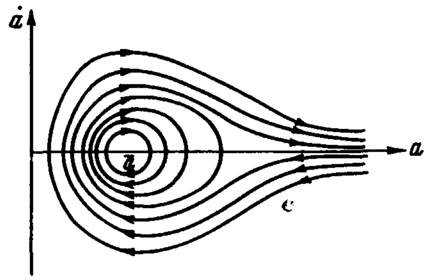

Fig. 2

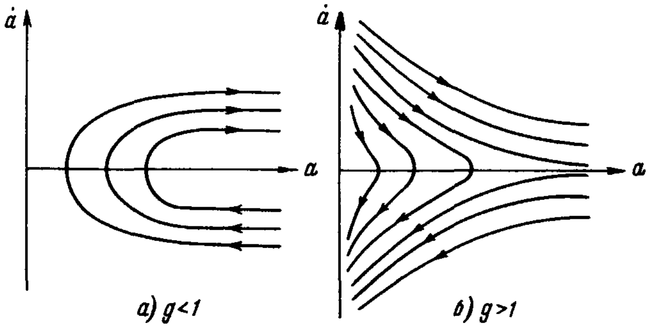

Fig. 3

The approximation (7) enables us to find a solution of Eq. (2) in the form of Gaussian beams

$$E_0 = E_M \exp\left(-\frac{x^2}{2a^2} - \frac{\varphi^2}{2b^2}\right)$$

By making the substitutions $x = \alpha a$ and $y = \beta b$ (α and β are constants for each ray tube) we then obtain from (4) equations for a and b. For a two-dimensional beam ($b \to \infty$), and also for a three-dimensional symmetrical beam ($a = b$), we get

$$\ddot{a} = a^{-3}[1 - g(a)] \qquad (8)$$

where $g = P/P_{st}$; P is the beam power, and $P_{st}(a)$ is the power of the stationary beam, characterized by the parameter a. The power of the two-dimensional beam is $P_{st}^{(2)}(a) = cn_0 \times (8\sqrt{\pi}B\epsilon' k^2 a)^{-1}$ (a is dimensional), while the power of the three-dimensional beam is $P_{st}^{(3)} = cn_0/8B\epsilon' k^2$ and is independent of its size. The values presented differ only by a numerical factor of the order of unity from the corresponding values for stationary beams that are rigorous solutions of (2). The phase portrait of Eq. (8) is shown in Fig. 2 for a two-dimensional beam and in Fig. 3 for a three-dimensional beam. $\bar{a}$ in Fig. 2 is the radius of the stationary beam. When $g < 1$ the three-dimensional beam always becomes defocused (Fig. 3a), and when $g > 1$ it is focused, under suitable initial conditions, at some point on the z axis (Fig. 3b), as in the geometric-optics approximation. The position of the focus depends on g. On the whole, the ray patterns for $g < 1$ and $g > 1$ are qualitatively the same as in Figs. 1a and 1b, respectively.

The behavior of the three-dimensional beam in the vicinity of focal points is not described by the foregoing equations. Phenomenologically, ideal focusing can be avoided in the three-dimensional case by assuming that $P_{st}^{(3)}$ begins to increase with decreasing a at sufficiently small values of a. In practice, however, the required beam defocusing action will occur at field intensities for which effects not accounted for here, nonlinear absorption of the beam energy and of the breakdown of the dielectric, become significant. In particular, the passage of a three-dimensional beam through a nonlinear dielectric can be accompanied by the formation of several sparks at the points of successive focusing of the beam.[2]

The author is grateful to A. V. Gaponov and M. A. Miller for useful discussions.

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[2] Chiao, Garmire, and Townes, Phys. Rev. Lett. 13, 479 (1964).
[3] N. G. Bondarenko and V. I. Talanov, Izv. VUZ'ov, Radiofizika 7, 313 (1964).

1) In the optical band the parameter ϵ' of many dielectrics has values 10^{-13} - 10^{-12} cgs esu [2].

2) S. B. Mochenev reported to the author that he observed such a phenomenon when an intense beam of light (λ = 1.06 μ) was focused in water and in carbon tetrachloride.

GENERATION OF ULTRAVIOLET RADIATION BY USING CASCADE FREQUENCY CONVERSION

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At the present time the only way to obtain intense radiation at wavelengths shorter than 0.7 μ is to use nonlinear-optics methods. We report here some results of an experimental investigation aimed at producing radiation sources in the 0.53 - 0.26 μ band, with output power not lower than 3 - 5 MW. To cover this band, we used cascade conversion of the frequency of a neodymium-glass laser. A block diagram of the experimental set-up is shown in the Figure. The radiation from the neodymium-glass laser, with λ_1 = 1.06 μ and with power P_1 (its resonator was Q-switched with a rotating prism), was subjected to successive nonlinear transformations in KDP or ADP crystals. All the transformations were made in unfocused beams. In the first KDP crystal (length l = 3 cm) the laser frequency was doubled (output wavelength λ_2 = 0.53 μ). The power P_2 of the harmonic was sufficient for further effective frequency conversion; this was effected either by another frequency doubling (in which case the output was the fourth harmonic of the fundamental radiation λ_4 = 0.265 μ, with power P_4), or by mixing the frequencies of the fundamental radiation and the second harmonics (thus generating the third harmonic of the fundamental radiation λ_3 = 0.353 μ, with power P_3). The lengths of the corresponding nonlinear crystals were 2 - 3 cm. Using stimulated Raman scattering at λ_1 or λ_2 (see [2]), we can obtain a set of discrete spectral lines, the distance from which to λ_3 or λ_4 is equal to the corresponding frequency of the molecular oscillations. The intensity of the Raman scattering lines was 5 - 10% of the