crystal (see [7]), is still lower than the efficiency of a cascade converter for a glass-laser radiation to the wavelength $\lambda_4 = 0.265\ \mu$.

In conclusion it must be emphasized that the power we obtained in the 0.53 - 0.26 μ range is quite sufficient for the registration of many nonlinear effects, particularly two-photon absorption (and possibly the corresponding recombination glow) in many dielectrics. The use of harmonic generators makes it possible to measure the frequency dependence of the nonlinear absorption. Preliminary experiments with KDP crystals have shown that 100 MW/cm^2 power at $\lambda_4 = 0.265\ \mu$ severely damages these crystals; at the same time, comparable power at $\lambda_2 = 0.53\ \mu$ produced no damage. An investigation of the cross sections of two-photon absorption in the ultraviolet region is now under way.

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1) Details of the theoretical analysis, carried out with V. G. Dmitriev, of the factors which determine the limiting values of η_2 will be published separately.

2) In principle, cumulative nonlinear effects can be obtained by using different phase-shift compensation schemes, such as proposed in [5,6], but their technical realization is rather complicated.

SUM RULES FOR THE COUPLING CONSTANTS G(B*, BP) IN BROKEN $\tilde{U}(12)$ SYMMETRY

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The $\tilde{U}(12)$ symmetry scheme proposed by Salam et al. [1] and constituting a relativistic generalization of SU(6) symmetry [2] makes it possible to write the S matrix of a strong interaction process in a relativistically invariant manner. In this scheme the baryons $3/2^+$ and $1/2^+$ are described by tensors satisfying the Bargmann-Wigner equation [3], and belong to the

364 multiplet, whereas the 0^- and 1^- mesons belong to the 143 multiplet.

In spite of its great success, this scheme is incompatible with the unitarity condition [4], so that it has become customary recently to introduce a "momentum spurion" [5] which transforms like a component of the 143 representation. Thus, the formal $\tilde{U}(12)$ symmetry is broken not only by the application of the Bargmann-Wigner equation, but also by the introduction of the spurion. The effect without the spurion is regarded as the zeroth approximation, and with the spurion as the next approximation.

The purpose of the present paper is to find relations between the coupling constants of the decays $3/2 \to 1/2 + 0^-$ in the aforementioned spurion theory.

In the $\tilde{U}(12)$ scheme, the baryons $3/2^+$ and $1/2^+$ are described by the symmetrical tensor[1]

$$\Psi_{ABC}(\hat{p}) = \sqrt{3}(2\sqrt{2})^{-1}m^{-1}[(p + m)\gamma_\mu C]_{\alpha\beta}D_{\mu\gamma,abc} + (2\sqrt{6})^{-1}m^{-1}\{[(\hat{p} + m)\gamma_5 C]_{\alpha\beta}\epsilon_{abs}N^s_{\gamma,c} + \text{cycl.}\} \quad (1)$$

and the 0^- and 1^- mesons by the tensor

$$\Phi^B_A(q) = \mu^{-1}[(q + \mu)(\gamma_5 P^b_a + \gamma_\mu V^b_{\mu,a})]^\beta_\alpha \quad (2)$$

where $A \equiv \alpha a$, $B \equiv \beta b$, $C \equiv \gamma c$; α, β, and γ are spinor indices; a, b, and c are unitary indices; m and μ are the average masses of the baryons and mesons, respectively.

In the zeroth approximation, i.e., without account of the spurion, we have a single expression for the matrix element

$$\bar{\Psi}^{CDB}(p')\Psi_{CDA}(p)\bar{\Phi}^A_B(q) \quad (3)$$

We see immediately that the relations obtained from (3) for the decay constants of interest to us do not differ at all from those obtained on the basis of the exact SU(3) symmetry [6].

Let us consider the problem in the next higher approximation, where the effect with the spurion is taken into account. We choose the spurion in the form

$$S = \hat{p} \otimes \lambda_8 \quad (4)$$

in which account is taken also of the violation of SU(3) symmetry.

The spurion leads to the following additional terms for the matrix element

$$\bar{\Psi}^{CDB}\Psi_{CDA}S^E_B\bar{\Phi}^A_E \quad (5)$$

$$\bar{\Psi}^{CDB}\Psi_{CDA}\bar{\Phi}^E_B S^A_E \quad (6)$$

and

$$\bar{\Psi}^{CDB}\Psi_{CEA}S^E_D\bar{\Phi}^A_B \quad (7)$$

Substituting (1), (2), and (4) in (3) and (5) - (7) we arrive at the following expression for the transition matrix element

$$M = \{a_0\epsilon^{bcs}\bar{N}^d_s D_{\mu,cda}\bar{P}^a_b + a_1\epsilon^{3cs}\bar{N}^d_s D_{\mu,cda}\bar{P}^a_3 + a_2\epsilon^{bcs}\bar{N}^d_s D_{\mu,cd3}\bar{P}^3_b + a_3(\epsilon^{bcs}\bar{N}^3_s + \epsilon^{b3s}\bar{N}^c_s)D_{\mu,c3d}\bar{P}^a_b\}q_\mu \quad (8)$$

From this, and from the general expression

$$G\bar{\Psi}(p')\Psi_\mu(p)q_\mu\varphi(q)$$

for the amplitude of the $3/2^+ \to 1/2^+ + 0^-$ decay, we can express the coupling constants $G(B^*, BP)$ in terms of a_i:

$$\begin{array}{lclcl}
(N^*,N\pi) & \equiv & G(N^*,N\pi) & = & a_0 \\
(N^*,\Sigma K) & \equiv & -\ G(N^*,\Sigma K) & = & a_0 + a_1 \\
(Y^*,N\tilde{K}) & \equiv & \sqrt{3}G(Y^*,N\tilde{K}) & = & a_0 + a_2 \\
(Y^*,\Lambda\pi) & \equiv & \sqrt{2}G(Y^*,\Lambda\pi) & = & a_0 + a_3 \\
(Y^*,\Sigma\pi) & \equiv & -\sqrt{3}G(Y^*,\Sigma\pi) & = & a_0 + a_3 \\
(Y^*,\Sigma\eta) & \equiv & -\sqrt{2}G(Y^*,\Sigma\eta) & = & a_0 + (2/3)a_1 + (2/3)a_2 + (1/3)a_3 \\
(Y^*,\Xi K) & \equiv & -\sqrt{3}G(Y^*,\Xi K) & = & a_0 + a_1 + a_3 \\
(\Xi^*,\Lambda\tilde{K}) & \equiv & \sqrt{2}G(\Xi^*,\Lambda K) & = & a_0 + a_2 + a_3 \\
(\Xi^*,\Sigma\tilde{K}) & \equiv & -\sqrt{2}G(\Xi^*,\Sigma\tilde{K}) & = & a_0 + a_2 + a_3 \\
(\Xi^*,\Xi\eta) & \equiv & -\sqrt{2}G(\Xi^*,\Xi\eta) & = & a_0 + (2/3)a_1 + (2/3)a_2 + (4/3)a_3 \\
(\Xi^*,\Xi\pi) & \equiv & -\sqrt{2}G(\Xi^*,\Xi\pi) & = & a_0 + 2\,a_3 \\
(\Omega\ ,\Xi\tilde{K}) & \equiv & -(\sqrt{2})^{-1}G(\Omega\ ,\Xi\tilde{K}) & = & a_0 + a_2 + 2\,a_3
\end{array}$$

From this we obtain the next eight relations for the coupling constants

	left side	right side	
$(Y^*,\Lambda\pi) = (Y^*,\Sigma\pi)$	0.93	0.90	(9)
$(N^*,N\pi) + (\Xi^*,\Xi\pi) = (Y^*,\Lambda\pi) + (Y^*,\Sigma\pi)$	1.87	1.83	(10)
$(\Xi^*,\Lambda\tilde{K}) = (\Xi^*,\Sigma\tilde{K})$	0.98	0.93	(11)
$(\Omega,\Xi\tilde{K}) + (Y^*,N\tilde{K}) = (\Xi^*,\Lambda\tilde{K}) + (\Xi^*,\Sigma\tilde{K})$	1.96	1.91	(12)
$(Y^*,\Lambda\pi) + (Y^*,\Sigma\eta) = (N^*,N\pi) + (\Xi^*,\Xi\eta)$	1.83	1.87	(13)
$(Y^*,N\tilde{K}) + (Y^*,\Lambda\pi) = (N^*,N\pi) + (\Xi^*,\Lambda\tilde{K})$	1.98	1.98	(14)
$(N^*,\Sigma K) + (Y^*,\Lambda\pi) = (N^*,N\pi) + (Y^*,\Xi K)$	1.79	1.82	(15)
$\frac{1}{2}[(Y^*,\Xi K) + (\Xi^*,\Lambda\tilde{K})] = \frac{1}{4}[3(\Xi^*,\Xi\eta) + (N^*,N\pi)]$	0.90	0.91	(16)

We have written out alongside each relation the values calculated from the data of [7] for its left and right sides (see the table), choosing for conveninece $(N^*,N\pi)$ equal to unity. We see that these values are well satisfied by the obtained sum rule (9) - (16) (the average difference is 2.5%).

It is also easy to see that the results include all the relations obtained on the basis of the broken SU(3) symmetry. Indeed, by combining (9) - (16) we obtain, for example, all the relations (9.1) - (9.7) of [8].

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SYMMETRY OF THE HYDROGEN ATOM

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In view of the success of the group approach to elementary-particle physics, questions connected with the symmetry of wave equations are presently attracting much attention.

Fock [1] and Bargmann [2] have shown that in a Coulomb potential the wave functions belonging to one level realize a finite-dimensional representation of a compact group O_4, which has been regarded as the symmetry group for this problem.

Barut et al. have shown in a recent paper [3] that the states of the discrete spectrum of the hydrogen atom form a basis of an infinite-dimensional representation of the deSitter algebra (4 + 1). As shown by Thomas [4], the basis of the infinite-dimensional representation of the deSitter group S is formed by matrix elements of representations of the compact group O_4 contained in it.

The purpose of the present paper is to show that the "symmetry group" of the hydrogen atom is the non-compact group O_6, the Lie algebra of which is the algebra D_3, and to present a simple construction showing that the functions belonging to the discrete spectrum form a single infinite-dimensional irreducible representation of this algebra.

Let $\varphi(x_1 \dots x_n)$ satisfy the equation

$$\hat{A}\varphi = 0 \tag{1}$$

where $\hat{A}$ is a linear differential operator. We define as the symmetry group of Eq. (1) the aggregate of operators $\hat{M}_\alpha$ forming an algebra closed against commutation and satisfying the condition

$$[\hat{A}\hat{M}_\alpha]\varphi = 0 \tag{2}$$

If φ is a solution, then $\hat{M}_\alpha\varphi$ is also a solution.

As shown by Fock[1], the eigenfunctions of the discrete spectrum of the hydrogen atom, in the momentum representation and in the variables ζ_i $(i = 1, \dots 4)$