

A. A. Chaban

Acoustics Institute, Moscow

Submitted 15 July 1965

The reported observation of an additional ultrasonic signal [1] following amplification of 10^7 cps transverse oscillations by carrier drift in a CdS crystal has aroused great interest.

The anomalous signal passed through the crystal at a velocity smaller by a factor 1.3 - 1.6 than the ordinary signal. This gave grounds for suggesting that a collective phonon wave was observed, similar to "second sound" in helium [1-3]. Very simple estimates show, however, that at room temperature and at the conductivities employed in [1], the elastic oscillations (with frequencies on the order of 10^{10} - 10^{11} cps) whose amplification can lead in accordance with [1-3] to the existence of "second sound" are damped much more by the lattice anharmonicity than they are amplified by the carrier drift. This circumstance was already noted in [4].

We shall show in the present note that the appearance of the anomalous signal can be interpreted as some diffraction effect, caused by the anisotropy of the amplification coefficient. This phenomenon is similar to birefringence, where a wave with anomalously low front-propagation velocity likewise exists; in our case, however, the anisotropy of importance is not that of the real but of the imaginary part of the wave number.

Assume that electrons move along the z axis with drift velocity v in a medium occupying the half-space $z > 0$, which is elastically isotropic. A plane ultrasonic wave, propagating in the medium at angle θ to the z axis, will grow [5,6] if $v \cos\theta > s$, where s is the speed of sound, and the amplification coefficient will depend on the angle θ . Confining ourselves for simplicity to a longitudinal wave, let us investigate the radiation produced in the given medium by an infinite plate situated at $z = 0$ and vibrating with frequency ω . Each point on the plane $z = 0$ can be regarded as a source of spherical waves, which are amplified as they propagate, with different amplification coefficients in different directions. The displacements produced at a receiver located a distance z from the radiating plane by the elements of a ring seen from the reception point at an angle θ are equal to

$$du = A \cos\theta z^{-1} \exp[i(\omega t - \frac{kz}{\cos\theta}) + \frac{\alpha(\theta)z}{\cos\theta}] \cdot dS$$

where A is a constant, α is the amplification coefficient, and

$$k = \omega s^{-1}; \quad dS = 2\pi z \tan\theta d(z \tan\theta)$$

Integrating with respect to θ and introducing the symbol $p = 1/\cos\theta$, we obtain

$$U(z) = 2\pi A \int_1^\infty z \exp[i(\omega t - kzp) + \alpha(p)zp] dp \quad (1)$$

Knowing the dependence of α on p , we can obtain the field (1).

We shall carry out a concrete calculation for the simplest case of a cubic crystal, assuming that the amplification is at the expense of the deformation potential [7]. We assume that a major role is played by traps with $\omega\tau < 1$, where τ is the relaxation time of the conduction electrons with respect to the traps (such an inequality was apparently satisfied in [1]; see [8-9]). Then we can readily find from [8-10] that $\alpha(\theta) \approx \alpha_0 = \text{const}$ when $\theta < \theta_0$ and $\alpha(\theta) \leq 0$ when $\theta > \theta_0$. In this case we obtain approximately

$$U(z) = D \left\{ \exp \left[i(\omega t - kz) + \alpha_0 z \right] - \exp \left[i(\omega t - \frac{kz}{\cos \theta_0}) + \frac{\alpha_0 z}{\cos \theta_0} \right] \right\} \quad (2)$$

where D is a constant. Thus, two elastic waves propagate simultaneously in the medium, with velocities s and $s \cos \theta_0$ respectively; the second wave produces the anomalous signal. The velocity difference is connected with the fact that in the anomalous wave energy is transported (with the speed of sound) at an angle θ_0 to the direction of the front. We see from (2) that under our assumptions the anomalous wave can have a much larger amplitude than the ordinary wave. This makes it especially interesting and desirable to set up an experiment with cubic crystals, where the assumptions of the theory are well satisfied. In the case of piezoelectric crystals, an account of the anisotropy of the elastic and especially of the piezoelectric properties is of course indispensable for quantitative deductions.

The velocity of the anomalous wave observed experimentally in [1] agrees qualitatively with the theory presented above. Unfortunately, the character of the dependence of this velocity on the carrier drift velocity is not discussed in the cited paper. We note that the conditions for the observation of two non-overlapping (ordinary and anomalous) pulses is of the form

$$\Delta t < \frac{L}{s} \left(\frac{1}{\cos \theta_0} - 1 \right) \quad (3)$$

where L is the length of the crystal and Δt is the pulse duration.

The phenomenon considered here can be given a very lucid interpretation. If we assume that the character of the amplification is such that practically the entire radiation from a point source in the medium is concentrated in a cone of vertex angle θ_0 , then each point with $z > 0$ will receive radiation only from a disc whose axis passes through the point of observation and which is seen from the latter at an angle θ_0 , just as if the radiation were that of a plane wave normally incident on an opaque screen and passing through a round hole in it. Accordingly, when condition (3) is satisfied, two signals should be observed at the reception point: the direct signal passing through the hole, and the signal diffracted by the edge of the hole. The corresponding calculation for the case of pulsed radiation from a disc is contained in [11].

In conclusion we note that a perfectly analogous phenomenon, the reception of two signals, will occur also if the damping coefficient has a suitable anisotropy and there is no amplification. We have thus established the existence of two waves, ordinary and anomalous, for the case when the imaginary part of the wave vector is anisotropic, both when waves are amplified and when they are attenuated.

Analogous phenomena should be expected also when waves of arbitrary type propagate in a medium in which the properties that govern the propagation of waves of this type exhibit anisotropy.

The author is grateful to Yu. L. Gazaryan, M. A. Isakovich, and I. A. Urusovskii for valuable advice and for a discussion of the results.

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INVESTIGATION OF THE STRUCTURE OF THE FRONT OF A STRONG MAGNETIC-SOUND WAVE IN A RAREFIED PLASMA

S. P. Zagorodnikov, L. I. Rudakov, G. E. Smolkin, and G. V. Sholin

Submitted 17 July 1965

This paper is devoted to an experimental investigation of the structure of the front of a strong magnetic-sound wave propagating in a rarefied plasma transverse to a magnetic field. In laboratory experiments, an essential factor for such waves is the nonstationarity of the wave motion.

A theoretical description of nonstationary magnetic-sound wave of finite amplitude is the subject of [1-3]. In [1] Adlam and Allen solved numerically the problem of unsteady motion of a magnetic piston in a rarefied plasma for two concrete time variations of the magnetic field on the plasma boundary:

$$H_n(t_n) = 1 + \alpha t_n \quad (1)$$

$$H_n(t_n) = 1 + \beta[1 - \exp(-\alpha t_n)] \quad (2)$$

Here $H_n = H/H_0$ is the magnetic field normalized relative to the constant field H_0 ; $t_n = t/\tau_{ei}$ is the time normalized relative to $\tau_{ei} = c [\text{mM}]^{1/2}/eH_0$, while α and $\beta = H_n(\infty) - 1$ are constants.

For $\alpha = 1$ and $\beta = 1$ the authors found the profile of the magnetic field in the plasma at certain values of t_n . They showed that in case (1) the magnetic-field front, which increases linearly on the plasma boundary, is transformed inside the plasma into an exponentially growing