DOUBLE BREMSSTRAHLUNG IN ELECTRON COLLISIONS

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In this article we present the results of a calculation of the cross section for the emission of two photons with arbitrary energy in electron-electron and electron-positron collisions. This question is of great interest in connection with colliding beam experiments. The emission of two soft photons was considered in [1], and the emission of the soft photon and one photon of arbitrary energy was considered in [2], where a formulation of the problem and a method for its solution are given. We use here the notation of [2].

We represent the cross section of the process in the form

$$d\sigma_{\omega_1\omega_2} = \frac{4\alpha^4}{(2\pi)^4[(p_1p_2)^2 - 1]^{1/2}} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \int \frac{d^4\Delta}{\Delta^4} K_{1\mu\nu}K_2^{\mu\nu}$$

$$K_{1\mu\nu}K_2^{\mu\nu} = c_1^{(2)}I_1^{(1)} + c_2^{(2)}I_2^{(1)} + (\Delta n)^{-2}[c_1^{(2)}\Delta^2 + c_2^{(2)}(\Delta p_1)^2 + c_3^{(2)}\Delta^4 + 2c_4^{(2)}\Delta^2(\Delta p_1)]I_3^{(1)}$$

$$- (\Delta n)^{-1}[(\Delta p_1)c_2^{(2)} + c_4^{(2)}\Delta^2]I_4^{(1)}$$

where the superior index of c_k or I_i corresponds to the number of the vertex. Expressing the coefficients $c_k^{(2)}$ in terms of $I_i^{(2)}$, we rewrite (1) in the form

$$d\sigma_{\omega_1\omega_2} = \frac{4\alpha^4}{(2\pi)^4[(p_1p_2)^2 - 1]^{1/2}} \int \frac{d^4\Delta}{\Delta^4} \sum_{i,k} \lambda_{ik} I_i^{(1)} I_k^{(2)} \tag{2}$$

The quantities $I_i^{(1)}$ and $I_k^{(2)}$ differ from those obtained in [2] [formulas (32) - (38)] by quantities of the order of ϵ^{-2}.

The coefficients of $I_k^{(2)}$ in the expression for $c_i^{(2)}$ are functions of invariant combinations of the vectors p, Δ, and n [see formula (11) in reference [2]]. Inasmuch as these vectors do not include the vectors $K_{1,2}$, these coefficients, and consequently the values of λ_{ik}, do not depend on $\omega_{1,2}$. From formulas (32) - (38) of [2] we see that the quantities $I_k^{(1,2)}$ are of the following order in ϵ: $I_1^{(1,2)} \sim I_2^{(1,2)} \sim 1$, $I_4^{(1,2)} \sim \epsilon$, and $I_3^{(1,2)} \sim \epsilon^2$; this order does not depend on $\omega_{1,2}$. Consequently, the order of each of the terms in the sum $\Sigma_{ik}\lambda_{ik}I_i^{(1)}I_k^{(2)}$ does not depend on $\omega_{1,2}$, and by virtue of the symmetry with respect to the emission of photons 1 and 2, both $\Sigma_{ik}I_i^{(1)}I_k^{(2)}$ and $\Sigma_{ki}I_k^{(1)}I_i^{(2)}$ are of the same order. Starting from the fact that in the limit as $\omega_2 \to 0$ the dominating contribution (with respect to ϵ^2) is made by the $I_3^{(2)}$ term [2], it is clear that the most important term in (2) is $\lambda_{33}I_3^{(2)}I_3^{(1)}$. Direct calculation yields $\lambda_{33} = 4 + O(\epsilon^{-2})$. Retaining in the expressions for $I_3^{(1,2)}$ the terms that are principal in ϵ^2, we obtain for the double bremsstrahlung differential cross section, accurate terms of order ϵ^{-2}:

$$d\sigma = \frac{8\alpha^4\epsilon^4}{(2\pi)^4\Delta^4}\left\{-\frac{1}{\kappa_3^2} + \frac{\Delta^2[1 + (1 - \frac{\omega_1}{\epsilon})^2] + 4(1 - \frac{\omega_1}{\epsilon})}{2\kappa_1\kappa_3} - \frac{(1 - \frac{\omega_1}{\epsilon})^2}{\kappa_1^2}\right\}$$

$$\times\left\{-\frac{1}{\kappa_4^2} + \frac{\Delta^2[1 + (1 - \frac{\omega_2}{\epsilon})^2] + 4(1 - \frac{\omega_2}{\epsilon})}{2\kappa_2\kappa_4} - \frac{(1 - \frac{\omega_2}{\epsilon})^2}{\kappa_2^2}\right\}\delta(p_1 + p_3 + k_1 + k_2$$

$$- p_1 - p_2)\frac{d^3p_3}{\epsilon_3}\frac{d^3p_4}{\epsilon_4}\frac{d^3k_1}{\omega_1}\frac{d^3k_2}{\omega_2} \qquad (3)$$

In the integration of the cross section (2) over the 4-vector Δ, it is convenient to go over to the covariant variables Δ^2, κ_3, and κ_4. The foregoing analysis shows that, accurate to terms of order ϵ^{-2}, $I_3^{(1)}$ depends only on κ_3, and $I_3^{(2)}$ only on κ_4, with the integration over these variables being carried out from $\omega_{1,2}/2(\epsilon - \omega_{1,2})$ to ∞. We then obtain

$$d\sigma_{\omega_1\omega_2} = \frac{16\alpha^4}{\pi}\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}\int\frac{d\Delta^2}{\Delta^4}\left\{(1 - \frac{\omega_1}{\epsilon})\Phi(\frac{\Delta^2}{4}) + \frac{\omega_1^2}{\epsilon^2}\Delta(\Delta^2 + 4)^{-1/2}\ln(\frac{\Delta}{2}\right.$$

$$\left. + [1 + \Delta^2/4]^{1/2})\right\}\left\{(1 - \frac{\omega_2}{\epsilon})\Phi(\frac{\Delta^2}{4}) + \frac{\omega_2^2}{\epsilon^2}\Delta(\Delta^2 + 4)^{-1/2}\ln(\frac{\Delta}{2} + [1 + \Delta^2/4]^{1/2})\right\}, \qquad (4)$$

$$\Phi(x^2) = \frac{1 + 2x^2}{x(1 + x^2)^{1/2}}\ln[x + (1 + x^2)^{1/2}] - 1$$

As $\omega \to 0$, the expressions in the curly brackets are equal to $\Phi(\Delta^2/4)$, i.e., to a quantity proportional to the probability of emission of a classical photon with transfer of momentum Δ to the electron, integrated over the photon emission angles. These expressions can therefore be regarded as a generalization of such a probability to include the case of photons of arbitrary energy. When Δ^2 is small this probability is proportional to Δ^2, so that small Δ^2 are insignificant in (3) and the lower limit of integration can be set equal to zero. The upper limit of integration with respect to Δ^2 is proportional to ϵ^2 and, by virtue of the convergence of the integral, it can be set equal to infinity. After integration we obtain the following final formula for the double bremsstrahlung cross section:

$$d\sigma_{\omega_1\omega_2} = \frac{8r_0^2\alpha^2}{\pi}\left\{(1 - \frac{\omega_1}{\epsilon})(1 - \frac{\omega_2}{\epsilon})[5/4 + 7/8\,\zeta(3)] + [(1 - \frac{\omega_1}{\epsilon})\frac{\omega_2^2}{\epsilon^2}\right.$$

$$\left. + (1 - \frac{\omega_2}{\epsilon})\frac{\omega_1^2}{\epsilon^2}](1/2 + 7/8\,\zeta(3)) + \frac{\omega_1^2\omega_2^2}{\epsilon^4}7/8\,\zeta(3)\right\}, \quad 7/8\,\zeta(3) = 1.052 \qquad (5)$$

Formula (5) is valid in the hardest part of the spectrum, when $\epsilon - \omega$ is of the order of unity. However, in view of the narrowness of the interval, this region apparently makes no finite contribution to the integral cross section.

The ratio of the double bremsstrahlung cross section in the region of hard photons to the cross section for two-quantum annihilation of an electron-positron pair takes the form

$$\frac{\delta\sigma_b}{\sigma_a} = 1.7 \times 10^{-4} \frac{\epsilon^2}{\ln 4\epsilon} \left(\frac{\delta\omega}{\omega}\right)^2 \quad (6)$$

(the energy is in MeV).

If the photon detectors have a reasonable energy resolution, these cross sections become equal at energies on the order of 1 BeV.

[1] V. N. Bayer and V. M. Galitsky, Phys. Lett. 13, 355 (1964).

[2] V. N. Baier and V. M. Galitskii, JETP 49, 661 (1965), Soviet Phys. JETP 22, in press

EFFECT OF MICROWAVE RADIATION ON THE ELECTRIC CONDUCTIVITY OF p-TYPE INDIUM ANTIMONIDE

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The effect of microwave radiation on the electric conductivity of indium antimonide was investigated by many authors, both in the USSR and abroad [1-4]. The authors of [1,2] investigated the variation of the electric conductivity of n-type indium antimonide in a constant magnetic field or in the absence of a field, at helium temperatures, under the influence of radiation in the millimeter radio band.

We observed a decrease in the dc electric conductivity under the influence of microwave radiation with density $P \sim 10^{-6} - 10^{-7}$ W/mm^2 in samples of single-crystal p-type indium antimonide with Hall carrier density from 7×10^{12} to 4×10^{14} cm^{-3}, Hall mobility $\mu = 2 \times 10^3 - 1 \times 10^4$ cm^2 V^{-1} sec^{-1}, and electric resistivity $\rho = 4 - 100$ ohm-cm in the microwave range $\lambda = 2 - 30$ mm at temperatures 77 - 150°K. The relative change in the electric conductivity was $\sim 10^{-5} - 10^{-6}$. We used samples of different dimensions: length 1 - 8 mm, width 1 - 4 mm, thickness 0.5 - 0.01 mm.

We measured the alternating voltage produced across the sample under the influence of microwave radiation modulated at audio frequency. The sample was mounted on a foamed-plastic plate in the center of the waveguide. The current was fed to the sample through leads insulated from the waveguide walls.

The sample was connected to a battery through the primary winding of the input transformer of the U2-1A measuring amplifier.

We used also an ordinary circuit for connecting photoresistances for maximum sensitivity. The signal from the amplifier was recorded with a two-beam electronic oscilloscope, the second channel of which was fed, through an analogous U2-1A amplifier, from the wavemeter detector (Fig. 1).

The microwave power flowed from a klystron generator (1) to the waveguide channel through a precision polarization attenuator (2) with controlled attenuation. Part of the power was diverted by a coupler (3) to the input of the wavemeter (4).