Neutrino experiments yielded the estimate $\sigma(\nu_\mu p \to \nu_\mu p) < 0.03\,\sigma(\nu_\mu n \to \mu^- p)$ [6]. According to (6), the (νp) scattering is essentially the result of the renormalization effects, and $\sigma(\nu_\mu p \to \nu_\mu p) \lesssim 0.01\,\lambda_\mu^2 \sigma(\nu_\mu n \to \mu^- p)$. The intensity of (νn) scattering should be larger by two orders of magnitude. Apparently λ_μ^2 and λ_e^2 are sufficiently small, for otherwise the decays $K^\pm \to \pi^\pm + \nu + \bar{\nu}$ and $K_1^0 \to \pi^0 + \nu + \bar{\nu}$, whose probabilities relative to the corresponding K_{e3} decay are equal to $2(\lambda_\mu^2 + \lambda_e^2)\cos^2\theta$, would have already been recorded. For the hyperon neutrino decays $\Sigma^+ \to p + \nu + \bar{\nu}$, $\Lambda \to n + \nu + \bar{\nu}$, etc. the analogous probability is equal to half the preceding one. Exact experimental estimates are very desirable here, for with decreasing λ an increase takes place in the intensity of the interaction with the electronic and muonic neutral currents, and it becomes possible to observe its admixture in the competing electromagnetic interaction in (e^+e^-) decays of nuclei, and in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ in (ep) and (μp) scattering (in the decays $\pi^0 \to e^+ + e^-$, $\mu \to e^+ + e^-$, and $\mu \to \mu^+ + \mu^-$, only the probability changes).

A verification of the inequalities $G_{ee}G_{\nu_e\nu_e} < G^2$ and $G_{\mu\mu}G_{\nu_\mu\nu_\mu} < G^2$ would decisively indicate the existence of some mechanism responsible for the interaction of only charged currents, for example the charged vector boson.

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CHARGE ASYMMETRY IN RADIATIVE DECAYS OF K- AND ρ-MESONS UPON VIOLATION OF CP-INVARIANCE

V. V. Solov'ev and M. V. Terent'ev

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Several mechanisms for CP-invariance violation have been proposed recently [1-3], in which charge asymmetry is expected to arise, in general, in meson $\pi^+\pi^-\gamma$ decays. In the present note we wish to point out a rather simple circumstance, namely that no large charge asymmetry will arise under any violation of CP-invariance in decays of this type, if a decay to $\pi^+\pi^-$ without emission of a photon is allowed. We present in connection with this statement several quantitative estimates.

From among known mesons with spin 0 and 1, only K_1^0 and ρ^0 decay into a $\pi^+\pi^-$ pair without any additional hindrances compared with the decay into $\pi^+\pi^-\gamma$. Let us consider the process $K_1^0 \to \pi^+\pi^-\gamma$. The amplitude of such a decay $M^{(k)}$ is equal to the sum of three amplitudes $M^{(k)} = M_b^{(k)} + M_e^{(k)} + M_m^{(k)}$, where

$$M_b^{(k)} = e\, g_k\, M_k \left(\frac{p_-\, \epsilon^{(\lambda)}}{k\, p_-} - \frac{p_+\, \epsilon^{(\lambda)}}{k\, p_+}\right), \tag{1}$$

$$M_e^{(k)} = e \cdot \frac{1}{M_k^3}\, a_k\, (kp,\ kq)\ F_{\mu\nu}\, p_\mu\, q_\nu\ , \tag{2}$$

$$M_m^{(k)} = e \cdot \frac{1}{M_k^3}\, b_k\, (kp,\ kq)\ \epsilon_{\mu\nu\alpha\beta}\, F_{\alpha\beta}\, p_\mu\, q_\nu \tag{3}$$

$M_b^{(k)}$ describes the emission of a bremsstrahlung quantum, while $M_e^{(k)}$ and $M_m^{(k)}$ describe direct emission of electric and magnetic quanta, respectively, from a region whose dimensions are $\sim 1/M_k$. Here p and q are the sum and the difference of 4-momenta of the positive and negative pions; $F_{\mu\nu}$ is the electromagnetic field tensor, k and $\epsilon^{(\lambda)}$ are the 4-vectors of the momentum and photon polarization. The functions $a_k(kp,\ kq)$ and $b_k(kp,\ kq)$ depend on odd powers of kq only when CP parity is not conserved. The decay $K_1^0 \to \pi^+\pi^-\gamma$ is described with good accuracy by the bremsstrahlung amplitude [4]. To obtain charge asymmetry, we take also into account the interference of M_b^k with the CP-odd contribution from $M_e^{(k)}$, retaining the first terms of the expansion in kq/M_k^2 (pions in the D state). Since we are not interested in the photon polarization, there is no interference between $M_b^{(k)}$ and $M_m^{(k)}$, and the amplitude of the $K_1^0 \to \pi^+\pi^-\gamma$ decay can be written in our approximation in the form

$$M^{(k)} = \left[1 + \frac{2i\delta_k}{M_k^\sigma}\,(kq)(kp_-)(kp_+)\right] e\, g_k\, M_k \left(\frac{p_-\epsilon^{(\lambda)}}{kp_-} - \frac{p_+\epsilon^{(\lambda)}}{kp_+}\right), \tag{4}$$

where $\delta_k = A'_k(0)/q_k$. Characterizing the charge asymmetry by means of the difference $(dw_+/d\omega) - (dw_-/d\omega)$ between the number of cases of $K_1^0 \to \pi^+\pi^-\gamma$ decay with $E_{\pi^+} > E_{\pi^-}$ and $E_{\pi^-} > E_{\pi^+}$ for a given photon energy ω, we obtain from (4)

$$\frac{dw_+}{d\omega} - \frac{dw_-}{d\omega} = -\frac{\alpha |g_k|^2}{8\pi^2}\ \mathrm{Im}\ \delta_k \left(\frac{m_k}{M_k}\right)^4 f_k(y), \tag{5}$$

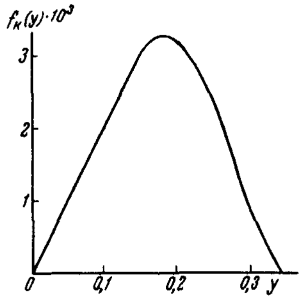
where

$$f_k(y) = y^2\,[2(y_k^{max} - y) + 4\epsilon^2_{(k)}\ \ln\,(4\epsilon^2_{(k)})(1 - 2y)]$$

and

$$y = \omega/m_k, \quad \epsilon y \epsilon y_k^{max}, \quad y^{max}_{(k)} = \frac{1}{2}\,(1 - 4\epsilon^2_{(k)}), \quad \epsilon_{(k)} = m_\pi/m_k \tag{6}$$

(m_π and m_k are the masses of the pion and K meson).



A plot of f_k against y is shown in the Figure. Integrating over the photon frequencies, we readily obtain

$$X_k = \frac{w_+ - w_-}{w_+ + w_-} = \mathrm{Im}\ \delta_k \left(\frac{m_k}{M_k}\right)^6 \frac{0.65 \times 10^{-3}}{[1.16\ \ln\,(m_k/\omega_{min}) - 2.7]}$$

$$= -\mathrm{Im}\ \delta_k \left(\frac{m_k}{M_k}\right)^6 \begin{cases} 0.15 \times 10^{-3} & \text{for } \omega_{min} = 1\ \text{MeV} \\ 0.35 \times 10^{-3} & \text{for } \omega_{min} = 10\ \text{MeV} \end{cases}, \tag{7}$$

where w_+ and w_- are the numbers of events with $E_{\pi^+} > E_{\pi^-}$ and $E_{\pi^-} > E_{\pi^+}$, respectively. ω_{min} is the experimental photon-frequency resolution. The dependence on ω in (6) can serve as an independent check on the approximation employed (on the dominance of the bremsstrahlung process). It is natural to expect that $M_k \sim m_k$. Thus, the charge asymmetry (formulas (6) and (7)) is very small even for strong violation ($\delta_k \sim 1$) of CP invariance in weak, strong, or electromagnetic interactions (see [2,3]). A large asymmetry would mean an unexpectedly large radius of the interaction region ($1/M_k$), and therefore an experimental investigation of the decay $K_1^0 \to \pi^+\pi^-\gamma$ is of particular interest. The charge asymmetry is determined by the imaginary part of δ_k, which results from the difference in the scattering phase shifts of the mesons in the P and D states. Let us consider the process $\rho^0 \to \pi^+\pi^-\gamma$. Its amplitude is $M^\rho = M_b^\rho + M_e^\rho + M_m^\rho$, where

$$M_b^\rho = eg_\rho\left(\frac{p_-\epsilon^\lambda}{kp_-} - \frac{p_+\epsilon^\lambda}{kp_+}\right) \Phi_\mu q_\mu , \tag{8}$$

$$M_e^\rho = \frac{ea_\rho}{M_\rho^2} F_{\mu\nu} \Phi_\mu p_\nu, \tag{9}$$

$$M_m^\rho = i \frac{b_\rho}{M_\rho^2} e F_{\mu\nu} \Phi_\mu q_\nu , \tag{10}$$

Φ_μ is the wave function of the ρ-meson, and the remaining symbols are the same as before [see (1) - (3)]. M_b^ρ and M_e^ρ are C-even amplitudes, corresponding to pions in states with $l = 0, 2, \ldots$ etc., while M_m^ρ is the C-odd amplitude ($l = 1, 3, \ldots$). Taking again into account only the interference between M_b^ρ and M_m^ρ (this approximation is apparently valid everywhere except on the upper end point of the photon spectrum), we obtain, for a specified photon frequency ω,

$$\frac{dw_+}{d\omega} - \frac{dw_-}{d\omega} = \frac{\alpha|g_\rho|^2}{24\pi^2} \left(\frac{m_\rho}{M_\rho}\right)^2 \mathrm{Im}\, \delta_\rho\, f_\rho(y), \tag{11}$$

where $\delta_\rho = b_\rho/g_\rho$ and b_ρ are assumed to be constant, while $f_\rho(y)$ is obtained from $f_k(y)$ by making the substitution $m_k \to m_\rho$, causing the maximum of the curve to move to the right.

Integrating over the photon frequencies, we get

$$X_\rho = \frac{w_+ - w_-}{w_+ + w_-} = \mathrm{Im}\, \delta_\rho \left(\frac{m_\rho}{M_\rho}\right)^2 \frac{6 \times 10^{-3}}{[1.78 \ln (m_\rho/\omega_{min}) - 3.87]}$$
$$= \mathrm{Im}\, \delta_\rho \left(\frac{m_\rho}{M_\rho}\right)^2 \begin{cases} 0.75 \times 10^{-3} & \text{for } \omega_{min} = 1 \text{ MeV} \\ 1.60 \times 10^{-3} & \text{for } \omega_{min} = 10 \text{ MeV} \end{cases} \tag{12}$$

Again, a study of the dependence on the frequency ω in (11) can serve as a check on the validity of the approximation employed. The remarks made in the discussion of formula (7) apply also to (12).

In conclusion let us discuss briefly the decays $K^\pm \to \pi^\pm\pi^0\gamma$. If CP-invariance is violated, the probabilities of these decays, and also the spectra of the positive and negative pions, may differ. The asymmetry should be of the order of $\delta M_d/M_b$, where δ is a parameter characterizing the degree of CP-parity nonconservation and M_d is the CP-odd part of the direct photon emission amplitude. Analogous remarks apply also to the decays $K^{*\pm} \to \pi^{\pm} K^0(\bar{K}^0)\gamma$ and $K^{*\pm} \to K^{\pm}\pi^0\gamma$. We note only that the difference in the form of the spectrum of the particles and antiparticles is large in the case of $K^{\pm}$ mesons, since M_b for $K^{\pm} \to \pi^{\pm}\pi^0\gamma$ has an additional degree of smallness, due to the rule BT = 1/2.

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ON THE MASSES OF PARTICLES (RESONANCES) WITH STRANGENESS S = -4 AND S = +1

Ya. B. Zel'dovich
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Harari and Lipkin [1] considered several properties of a hypothetical baryon 35-plet, which according to the SU(3) symmetry contains particles with strangeness from S = -4 (Y = -3) up to S = +1 (Y = +2).

In the quark model, this supermultiplet differs in the fact that it is made up of four quarks and one antiquark (see the Table below). We should therefore expect a non-monotonic variation of the particle mass as a function of the strangeness S or hypercharge Y. In fact, in the 35-plet the excited nucleon state [1)] with isospin $5/2^-$, N_5^* is made up of quarks, such as $4p$, $\bar{n}$; $3pn$, $\bar{n} + 4p$, $\bar{p}$; ..., $4n$, $\bar{p}$, i.e., without participation of strange quarks and antiquarks λ and $\bar{\lambda}$.

The state X_1 (S = -4, I = 1/2) is constructed like $4\lambda\bar{p}$; $4\lambda\bar{n}$ and it is natural to assume that X_1 is heavier than N_5^*, just as Ω is heavier than Δ in the decuplet, and just as Ξ is heavier than N in the octet; an intuitive common cause is the assumption that λ is heavier than n and p.

Thus, we expect a normal dependence of the mass on S or Y in the series $N_5 \ldots X_1$.

Let us turn to the state I_4 (S = +1, I = 2), which in terms of quarks is represented by $4p\bar{\lambda}$; . . .; $4n\bar{\lambda}$.

If λ is heavier than p and n, then $\bar{\lambda}$ is also heavier than $\bar{p}$ and $\bar{n}$ and we can therefore expect I_4 to be heavier than N_5^*; consequently, the quark model predicts here for the mass a strangeness dependence opposite from that which takes place in the octet and decuplet of baryons (but similar to the situation in mesons). However, such a situation does not contradict the existing concepts concerning mass splitting.

Within the framework of the Gell-Mann--Okubo formula

$$M = a + bY + c[I(I + 1) - 1/4\, Y^2] \qquad (1)$$