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## POSSIBLE CONNECTION BETWEEN THE AMPLITUDES OF THE PROCESSES $e^+e^- \to 3\pi$, $\gamma\gamma \to 3\pi$, and $\pi^0 \to 2\gamma$

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We consider in the single-photon approximation an invariant matrix element (ME) of the process $e^+e^- \to 3\pi$ (see Fig. 1) near the threshold

$$+ \frac{e}{q^2} \bar{v}(k')\gamma_\nu u(k)\, J_\nu(p_1, p_2, p_3), \tag{1}$$

where

$$J_\nu = \langle p_1 a, p_2 b, p_3 c \,|\, j_\nu(0) \,|\, 0\rangle = ih\epsilon_{abc}\epsilon_{\nu\alpha\beta\gamma} p_{1\alpha} p_{2\beta} p_{3\gamma}. \tag{2}$$

Here a, b, and c are the isotopic indices of the pions, $j_\nu$ is the electromagnetic current of the hadrons ($j_\nu = (1/2)e\bar{\psi}(1 + \tau_3)\gamma_\nu\psi + \ldots$). We assume that h is constant, a reasonable assumption near threshold.

We consider further the ME of the decay $\pi^0 \to 2\gamma$ (see Fig. 2):

$$M_{\nu\mu} = f\epsilon_{\nu\mu\alpha\beta}\, q_{1\alpha} q_{2\beta}, \tag{3}$$

where f is connected with the lifetime of the $\pi^0$ meson: $\tau = 64\pi/f^2\mu^3$, where $\mu$ is the pion mass. Under very likely assumptions, we shall show further that

$$h = f/eF_\pi^2, \tag{4}$$

where $F_\pi = 0.93\mu/\sqrt{2}$ is the constant of the $\pi^+$-meson.

The derivation of relation (4) follows, strange as it seems, from a consideration of the properties of the more complicated amplitude $R_{\nu\mu}$, which describes the process $\gamma\gamma \to 3\pi$ (see Fig. 3a):

$$R_{\nu\mu}^{abc} = i\int dx e^{-iq_1 x} \langle p_1 a, p_2 b, p_3 c \,|\, T(j_\nu(x) j_\mu(0)) \,|\, 0\rangle = R_{\nu\mu}^{pole} + R_{\nu\mu}^{cont}. \tag{5}$$

After explicitly separating the pole terms ($R_{\nu\mu}^{pole}$), the remaining contact part of the amplitude ($R_{\nu\mu}^{cont}$) should be expanded in powers of the momenta $p_i$ and $q_j'$. Confining ourselves to the lowest terms of the expansion and taking into account the requirements of G parity and Bose symmetry, we find that the contact part is equal to:

$$R_{\nu\mu}^{cont} = \epsilon_{\nu\mu\alpha\beta}(-A(q_1 - q_2)_\alpha p_{1\beta} + Bq_{1\alpha}q_{2\beta})\delta_{3a}\delta_{bc} + \text{permut}: (p_1, a \to p_2, b \to p_3, c), \tag{6}$$

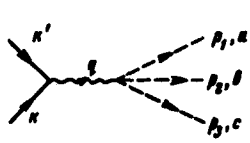


Fig. 1

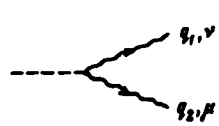


Fig. 2

where A and B are certain constants. The pole part of the amplitude (5) is equal to the sum of the diagrams of Fig. 3b and Fig. 3c. The pole term in Fig. 3b includes the amplitude (3) and the $\pi\pi$ scattering amplitude, which

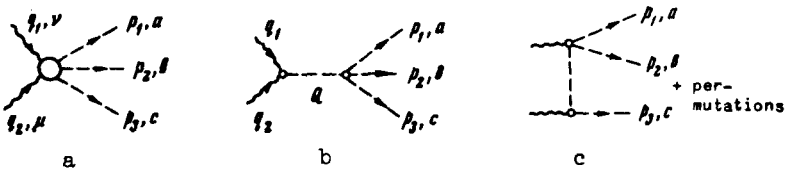


Fig. 3

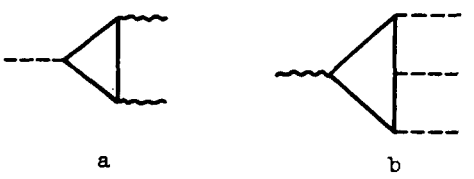


Fig. 4

is equal to (see [1]):

$$F_\pi^{-2}[(Q-p_1)^2-\mu^2]\,\delta_{3a}\delta_{bc}\ +\text{permut}:(p_1,a\to p_2,\ b\to p_3,\ c). \tag{7}$$

The pole term 3c includes the amplitude (2) and the charge vertex of the pion. The condition of gauge invariance connects the constant A in (6) and the constant h in the diagram of Fig. 3c (the diagram 3b is gauge-invariant in itself). It then turns out that

$$A = eh. \tag{8}$$

To obtain additional limitations, we consider separately the two amplitudes in (5): $R_{\nu\mu}^{311}$ (describing the process $\gamma\gamma \to \pi^0\pi^+\pi^-$) and $R_{\nu\mu}^{333}$ describing the process $\gamma\gamma \to 3\pi^0$). The condition for the partial conservation of the axial current (PCAC) for the neutral meson with momentum $p_1$ gives the relation $R_{\nu\mu}^{311} \to 0$ as $p_1 \to 0$. This enables us to determine the constant B in (6) in terms of the contribution of the diagram 3b as $p_1 \to 0$ (the contribution of the diagram 3c in (5) vanishes itself as $p_1 \to 0$). It then turns out that

$$B = f/F_\pi^2\ . \tag{9}$$

Thus, we have already expressed the entire amplitude $R_{\nu\mu}$ in terms of parameters measured in other processes. Using in addition the PCAC condition for one of the neutral pions in the $R_{\nu\mu}^{333}$ amplitude, we find that $R_{\nu\mu}^{333} \to 0$ as $p_i \to 0$ ($i = 1, 2, 3$). Under these conditions the pole term in Fig. 3c is equal to zero and the pole term in Fig. 3b is equal to $(-f/F_\pi^2)\varepsilon_{\nu\mu\alpha\beta}q_{1\alpha}q_{2\beta}$, and the contact term (6) is equal to $(-2A + 3B)\varepsilon_{\nu\mu\alpha\beta}q_{1\alpha}q_{2\beta}$. Equating to zero the summary contribution of the pole and contact terms in $R_{\nu\mu}^{333}$ and taking (8) and (9) into account, we obtain relation (4).

Relation (4) was verified by us in the $\sigma$ model with the aid of a calculation of the diagrams in Fig. 4a and Fig. 4b. The expression for $R_{\nu\mu}$ in the form (5) and (6), at values of A and B from (8) and (9) was also confirmed by a calculation of the diagrams in the $\sigma$ model. Relation (4) was obtained by us also with the aid of a direct application of PCAC in succession to each of the three pions in the amplitude (2) with allowance for the Schwinger terms resulting from the contribution of the singular triangular diagram.

We emphasize that the possibility of applying the PCAC with respect to one neutral meson in the amplitude $R_{\nu\mu}$ (which we have used in the text) is justified and is confirmed by perturbation theory, since $R_{\nu\mu}$ is described by well-converging ("five-point") diagrams.

A naive application of PCAC to the amplitude (3) leads, as is well-known (see [2, 3]) to a hindrance on the $\pi^0 \to 2\gamma$ decay, namely, it turns out that

$f \sim (q_1 + q_2)^2$ and the amplitude (3) contains in fact terms of fourth order in the momenta (the amplitude (2) contains in this case respectively terms of fifth order in the momenta). If the decay is actually forbidden within the framework of PCAC, then all our reasoning is incorrect (particularly, it is necessary to take into account in (6) fourth-order terms in the momenta and no relations can be obtained at all).

It is known, however, that the constant f is not small and has a "normal" order of magnitude ($f \simeq (1.4/\mu)(\alpha/\pi)$) and is given with accuracy of $\sim 1\%$ (!) in the $\sigma$ model by the contribution of only one triangular diagram with a nucleon loop (see Fig. 4a). We therefore regard as very likely the point of view formulated in [4], where it is noted that owing to the singular character of the triangular diagram the requirement of gauge invariance makes it necessary to modify the relations for the divergence of the axial current, which is manifest in other words in anomalous Schwinger terms in the current commutators. The $\pi^0 \to 2\gamma$ decay is then allowed and is due only to the contribution of the diagram of Fig. 4a. Such a picture, however, is not an unambiguous consequence of the Lagrangian formalism in some model (see, e.g., [5]). If relation (4) is confirmed, this will be a serious argument in favor of such a picture.

A verification of (4) is possible even now by a study of the process $e^+e^- \to \gamma \to 3\pi$. An expression for the cross section of this process and the corresponding numerical estimates are contained in [6].

A verification of (8) and (9) is possible in principle by a study of the process $ee \to ee + 3\pi$. The corresponding cross sections can be estimated on the basis of the formulas from [7] and turn out to be of the order of $\sim 10^{-35}$ cm$^2$.

In conclusion we note that the $\gamma\gamma \to 3\pi$ process was investigated recently in [8]. The conclusions of [8] are incorrect, since the authors did not consider the more general structure of the amplitude $R_{\nu\mu}$.

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## ERRATUM

The initials of two of the authors of "Influence of Ultrasonic Excitations in Crystals on the Probability of the Mossbauer Effect," Vol. 13, No. 10, p. 388, should be T. M. (not G.M.) Aivazyan and L. A. (not R.N.) Kocharyan.