amplitude to the corresponding extremum of the magnetoresistance (see Fig. 1). Then in our case this ratio is $\delta = 1.25$ at a measuring current $I_{meas} = 100$ mA. With increasing measuring current, the deviation from sinusoidal behavior decreases. Thus, at $I_{meas} = 3$ A we have $\delta = 1.15$. The decrease of the deviation with increasing current is probably due to the increase of the inhomogeneity of the magnetic field over the cross section of the sample, which may cause a smearing of the domain structure of the sample.

The influence of the measuring current on the measurement results is manifest also in the oscillation amplitude. Thus, for example at $I_{meas} = 100$ mA the oscillation amplitude is $\rho_{osc}/\rho = 0.79$ whereas at $I_{meas} = 3$ A we have $\rho_{osc}/\rho = 0.70$, i.e., an increase of the current to 3 A decreases the oscillation amplitude by 12%.

It can be concluded on the basis of these investigations that the carrier effective mass m*/m determined from the temperature dependence of the oscillation amplitude and the breakdown parameter H_0 are in satisfactory agreement with the data obtained by other methods.

The observed dependence of the amplitude and of the form of the oscillations on the measurement current is apparently determined by the change of the inhomogeneity of the magnetic field, and can be reconciled with the results of [7].

The observed dependence of the magnetic-breakdown probability on the field may apparently be the consequence of the coherent effects considered, for example, in [7].

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COSMIC RAYS AND THE ELEMENTARY LENGTH

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Much attention has been paid of late to the disparity between the theoretical estimates and the experimental data on primary cosmic-ray particles with energy $E_p > E_0 \sim 5 \times 10^{19}$ eV. The spectrum of such particles should be cut off at $E_p \sim E_0$, owing to their deceleration by residual photons (T $\sim$ 3°K) [1], something not observed in the experiments [2].

There is apparently still no undisputed explanation of this disparity. This raises the question (posed to the authors by G.B. Khristiansen) whether the violations of fundamental principles, expected by many at high energies, do not manifest themselves under the conditions in question. At first glance, the answer should be in the negative. The point is that the energy of the "photon-photon" system in the c.m.s. is only of the order of 10 MeV at $E_p \sim E_0$ (after subtracting the rest mass). This corresponds to a characteristic length $\ell \sim 10^{-12}$ cm. Yet experiments aimed at verifying quantum electrodynamics and the dispersion relations (EV for short) confirm the applicability of the indicated principles down to lengths 10^{-14} - 10^{-15}. The length ℓ is therefore still far from the "elementary length" at which the violations in question might appear.

However, the situation under consideration is not typical in the sense that it corresponds to a unique Lorentz factor $\gamma = E_p/M_p c^2 > 5 \times 10^{10}$, larger by many orders of magnitude than under the EV conditions. At the same time the quantity γ, which connects the "earth" reference frame (e-system), where the photon distribution is measured, and the proton rest system (ℓ-system), where the photoabsorption cross section is measured, enters significantly in the calculation of the spectrum cutoff. Importance attaches in this respect to the statistical factor $H = \exp(-\omega_e/kT)$, where ω_e is the photon energy in the e-system, corresponding to the characteristic photoabsorption energy $\omega_\ell \sim 100$ MeV in the lab system; for frontal collisions $\omega_e = \omega_\ell/2\gamma$ [1].

We can therefore expect that an affirmative answer to the question raised above is still possible without contradicting the EV results. This is confirmed by a model introduced below, in which the usual relativistic relations are violated at sufficiently large γ.

Relegating the details to a special article, we point out that this model is constructed by replacing the usual invariant $\dot{I}_0 = E^2/c^2 - p^2$ by a more complicated expression for $\dot{I}$, the invariance condition of which gives a generalization of the Lorentz transformation, and the equation $\dot{I} = M^2c^2$ gives the dynamics of the particle. The requirements that $\dot{I}$ go over into $\dot{I}_0$ at small values of the parameter $\xi = p_2/\dot{I}_0 \simeq \gamma^2 - I$ (which has the same form in all the reference frames [3]!) and that the law of dispersion of light be the usual one, yield

$$\dot{I} = E^{*2}/c^2 - p^{*2},$$
$$E^*/E = p^*/p = f(\xi), \quad f(\xi) = 1 - \alpha\xi^2 + \ldots \quad (\alpha\xi^2 << 1). \tag{1}$$

For light, $\xi \to \infty$, $f(\infty) = \text{const}$, and $\dot{I} = f^2(\infty)\dot{I}_0$. In this model, the principles of relativity and fundamentality of the speed of light are satisfied, but the symmetry between energy and momentum is violated.

The transformation law of E and p is determined by the usual Lorentz transformations for E* and p*; however, the transformation parameters does not coincide with the relative velocity. Comparing the values of the invariant $E^*\omega^*/c^2 - p^*k^*$ in the e- and ℓ-systems, we obtain for frontal collisions $\omega_e = \omega_\ell/2\gamma f(\xi)$. At $\alpha\xi^2 << 1$ but $\xi >> 1$ and $\xi \simeq \gamma^2$, the statistical factor is

$$H = \exp(-\omega_\ell/2kT\gamma)\exp(-\alpha\omega_\ell\gamma^3/2kT). \tag{2}$$

When $\alpha > 0$, the second factor strongly suppresses the deceleration of the proton and counteracts the spectrum cutoff (at $\alpha\xi^2 > 1$ this effect takes place if $f(\infty) < 1$). Stipulating that this factor be of the order of unity for $E_p \sim E_0$

(this is necessary to avoid a kink in the spectrum) we get $\alpha \sim 10^{-44}$. The exceptional smallness of this quantity causes the ordinary theory to be suitable under the EV conditions with a large margin.

The dimensionless constant α is strikingly close to the coupling constant of the quantum theory of gravitation $Km^2/\hbar c = 2 \times 10^{-45}$, referred to the mass of lightest particle, the electron (K is the gravitational constant). It is possible that this coincidence is not accidental, all the more since there are other empirical indications that the characteristic length of the quantum theory of gravitation $(K\hbar/c^3)^{1/2} \sim 10^{-33}$ may play the role of the elementary length [4].

The foregoing considerations, of course, do not pretent to be a real solution of the problem of the cosmic-ray spectrum at ultrahigh energies. Nonetheless, they show that in the case when less radical paths are exhausted, there remains in principle the possibility that new physical laws may appear under these conditions.

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THERMODYNAMIC QUANTITIES NEAR THE CRYSTALLIZATION POINTS OF LIQUIDS

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In connection with our general point of view [1], we expect identical singularities of the thermodynamic quantities at different points of phase transitions with change of symmetry, if these quantities are considered as functions of correctly chosen variables. Since the type of transition is of no importance for this point of view, it seems possible to apply this statement also to thermodynamic quantities near first-order transitions with change of symmetry.

It has been shown many times [2 - 4] that in such transitions there exists besides the δ-function in the specific heat, corresponding to the presence of latent heat Q_{lat}, also a trace of a genuine singularity that becomes cut off in the immediate vicinity of the transition with decreasing Q_{lat}. The presence of Q_{lat} and the jump of the ordering parameter are, as it were, an external limitation that disrupts the development of the singularity near the isomorphic phase-transition point. Expectation of this fact has led us to pay special attention to the crystallization point of liquids. This is a first-order phase transition with change of symmetry, and is characterized by relatively small latent heat (compared, say, with the binding energy).

Most experimental data point to only a small change of the specific heat near melting point, on the side of the liquid phase. However, the thermodynamic quantity possessing a singularity at the transition points in accord with [1] is not the specific heat $C = T(\partial S/\partial T)$, but the derivative of the entropy $\partial S/\partial T$,