

(this is necessary to avoid a kink in the spectrum) we get $\alpha \sim 10^{-44}$. The exceptional smallness of this quantity causes the ordinary theory to be suitable under the EV conditions with a large margin.

The dimensionless constant α is strikingly close to the coupling constant of the quantum theory of gravitation $Km^2/\hbar c = 2 \times 10^{-45}$, referred to the mass of lightest particle, the electron (K is the gravitational constant). It is possible that this coincidence is not accidental, all the more since there are other empirical indications that the characteristic length of the quantum theory of gravitation $(K\hbar/c^3)^{1/2} \sim 10^{-33}$ may play the role of the elementary length [4].

The foregoing considerations, of course, do not pretend to be a real solution of the problem of the cosmic-ray spectrum at ultrahigh energies. Nonetheless, they show that in the case when less radical paths are exhausted, there remains in principle the possibility that new physical laws may appear under these conditions.

The authors thank G.B. Khristiansen and V.N. Sazonov for valuable discussions.

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THERMODYNAMIC QUANTITIES NEAR THE CRYSTALLIZATION POINTS OF LIQUIDS

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Submitted 30 June 1971

ZhETF Pis. Red. 14, No. 4, 263 - 266 (20 August 1971)

In connection with our general point of view [1], we expect identical singularities of the thermodynamic quantities at different points of phase transitions with change of symmetry, if these quantities are considered as functions of correctly chosen variables. Since the type of transition is of no importance for this point of view, it seems possible to apply this statement also to thermodynamic quantities near first-order transitions with change of symmetry.

It has been shown many times [2 - 4] that in such transitions there exists besides the δ -function in the specific heat, corresponding to the presence of latent heat Q_{lat} , also a trace of a genuine singularity that becomes cut off in the immediate vicinity of the transition with decreasing Q_{lat} . The presence of Q_{lat} and the jump of the ordering parameter are, as it were, an external limitation that disrupts the development of the singularity near the isomorphic phase-transition point. Expectation of this fact has led us to pay special attention to the crystallization point of liquids. This is a first-order phase transition with change of symmetry, and is characterized by relatively small latent heat (compared, say, with the binding energy).

Most experimental data point to only a small change of the specific heat near melting point, on the side of the liquid phase. However, the thermodynamic quantity possessing a singularity at the transition points in accord with [1] is not the specific heat $C = T(\partial S/\partial T)$, but the derivative of the entropy $\partial S/\partial T$,

which is equal to C/T . This difference is of no significance in the immediate vicinity of the transition point, where T is almost constant, but is important on the tail of the singularity.

We have analyzed the behavior of this quantity for a great variety of substances [5 - 7] as a function of the dimensionless temperature T/θ in a wide range of values¹⁾. The region of singularity (Fig. 1) adjacent to the melting point was distinctly delineated. At the melting point itself, the specific heat is finite, something naturally connected with the termination of the transition at the point of absolute instability of the liquid phase T_0 .

The ordinates represent the specific heats C_p , C_σ , and C'_v of different substances. It is clear that because of the small compressibility of the liquid near the melting point, the plots of all these quantities are approximately the same. The true quantity characterizing the singularity would be the specific heat at constant zero value of ordering, which is possibly closest to C_v . Since the

temperature T_0 lies below the melting point and cannot be determined directly from experiment, it becomes an arbitrary parameter which is chosen to fit the approximation of the data by means of some definite relation. Figure 2 shows plots of C/T of several substances against $\tau^{-\alpha} = [(T - T_0/\theta)^{-\alpha}]$, which show that this singularity admits of an approach in the sense of "scaling" [9], like the critical point. Although the accuracy of the employed data is such that the value of the index α can be estimated only with an error of 100% (i.e., $\alpha \approx 0.13 \pm 0.13$), we shall use the value $\alpha = 0.13$, just as at the critical point [10], in order not to introduce any new constants. We see from Fig. 2 that all substances have regions where the "scaling" relation $C/T = A\tau^{-\alpha} + B$ is valid and that the coefficients A are not universal.

There are indications that the value of T_0 for water [11] shifts with increasing pressure. This should be the case for spinode points. This raises the hope that at the triple point of water, which is realized at a pressure $p = 2047$ atm [11], the spinodal intersects the melting curve and the corresponding point is an isolated critical Landau point [12] ($T_m \rightarrow T_0 \rightarrow T_{tr}$). It is

$$p \rightarrow p_{tr}$$

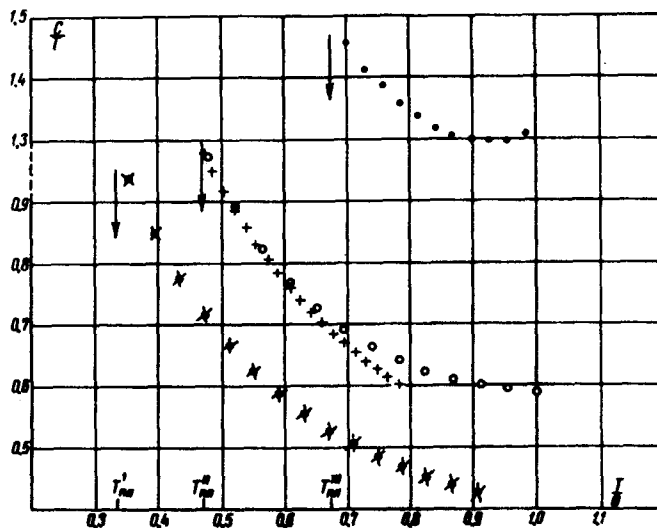


Fig. 1. Temperature dependence of the specific heat for different substances in the vicinity of the melting point. \circ - C'_v/T of oxygen, $+$ - C_p/T of oxygen, \bullet - C_σ/T of neon, \ast - C_p/T of propane. C'_v - specific heat at constant volume of the two phase system, and is close to the specific heat C_σ of a liquid in equilibrium with its saturated vapor. The ordinate scale for Ne has been shifted.

¹⁾The temperature is made dimensionless here with the aid of the de Boer molecular units $\theta = \epsilon/k$, where ϵ is the constant of the Lennard-Johns potential and k is Boltzmann's constant [8]. It is possible, however, to refer the temperature also to units of T_c .

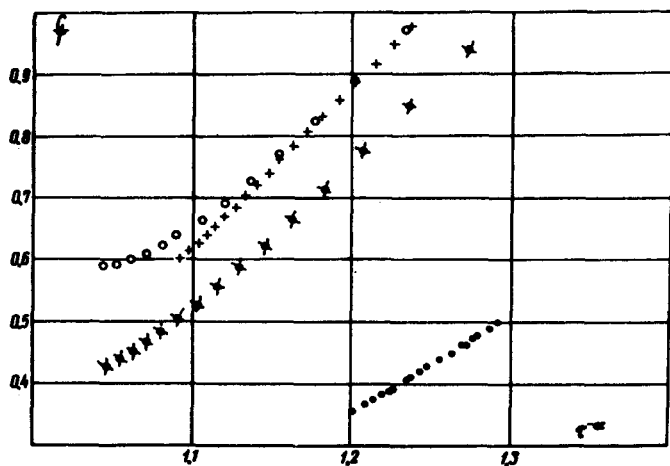


Fig. 2. Plot of $(\partial S/\partial T)$ for several substances against $\tau^{-\alpha}$ in the vicinity of the melting point. o - C'_V/T of oxygen, $T_0 = 32^\circ\text{K}$, $\theta = 115^\circ\text{K}$; + - C_p/T of oxygen, $T_0 = 32^\circ\text{K}$, $\theta = 115^\circ\text{K}$; * - C_p/T of propane, $T_0 = 50^\circ\text{K}$, $\theta = 254^\circ\text{K}$; ● - C_p/T of mercury, $T_0 = 120^\circ\text{K}$, $\theta = 851^\circ\text{K}$, $\tau = (T - T_0)/\theta$. It is obvious that the characteristic temperature θ of mercury, and possibly of other substances, is arbitrary, but the choice of θ does not affect the form of the curve.

thermodynamic function, and this contribution decreases with the temperature.

It can also be stated that near the melting point, on the side of the liquid phase, an increase takes place in the radius of correlation of the ordering, usually called the short-range order [16], which is the analog of the density correlation radius characterizing the critical point [9]. We can thus state in summing that whereas it is impossible to construct a theory of melting on the basis of [4], at any rate a correct approach to the liquid state is ensured not so much by an analysis of this state on the basis of the quasisolid model [15], as by the theory of scale transformations [9]. The liquid state is always within the limits of slight deviations from T_0 or T_c .

In conclusion, I wish to thank V.V. Shchekochikhin for the computations.

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possible that the maxima on the curves of many substances [13] are also points where the spinodals approach the melting curve.

Of course, besides the singularity of the specific heat, there is also corresponding anomalies of other quantities near the crystallization point. Thus, for many substances (e.g., water) the temperature dependence of the density has an anomalous character near the melting point. It was recently observed that $(\partial\rho/\partial T)$ of alkali metals changes noticeably near the melting point [14], and in glycerine this quantity has an appreciable anomaly below the melting point in the region of the supercooled state [15].

Since the corresponding quantities remain finite, the breakdown into regular and singular parts is ambiguous and leads to doubtful interpretations.

It is important to emphasize, however, that near the melting point there is a noticeable contribution of the fluctuations of the ordering in the

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CRITICAL CHARGE IN COLLISIONS OF NUCLEI

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Submitted 5 July 1971

ZhETF Pis. Red. 14, No. 4, 267 - 270 (20 August 1971)

The critical charge Z_c is defined as that value of the nuclear charge at which the energy of the ground level of the electron reaches the limit of the lower continuum $\epsilon = -1$ ($\hbar = c = m_e = 1$). At $Z > Z_c$ the Coulomb field of the bare nucleus¹⁾ produces two electron-positron pairs, the electrons of which settle on the 1S level, and the positrons go off to infinity through the Coulomb barrier (a detailed discussion of the properties of the stable system produced - the supercritical atom - produced after the positron emission can be found in [1]). According to calculations [2], $Z_c = 170$ for the isolated nucleus. Therefore, in spite of the latest progress in searches for superheavy elements [3, 4], the possibility of the existence of nuclei with $Z > Z_c$ appears at present to be purely hypothetical.

Apparently a more realistic method of verifying the theory of supercritical atoms [1, 5] is to observe spontaneous quasistatic production of positrons upon collision of heavy nuclei, say two bare uranium nuclei. The idea of such an experiment is that when two nuclei come close together to a distance $R < \hbar/m_e c = 1$ the electron is acted upon by a field analogous to the Coulomb field of a nucleus with double the charge $2A$. Therefore, for example, in the limiting case $R = 0$ (coalescence of the nuclei) the critical charge $Z_c(R)$ is decreased by one-half, $Z_c(0) = (1/2)Z_c(\infty) = 85$. For quantitative predictions it is necessary to calculate the dependence of Z_c on the distance between the nuclei R . In view of the large mass of the nuclei, they can be regarded as being at rest (all the more since the probability of positron production increases strongly with decreasing R , and at the instant of closest approach of the nuclei it vanishes). As shown in [6], to solve this problem one can use a variational method. We present below preliminary results of such a calculation.

The motion of the levels with increasing Z , the dependence of Z_c on the radius of the nucleus, and other characteristic features of the relativistic

¹⁾I.e., of a nucleus with all its electrons removed. Incidentally, for the production of positrons at $Z > Z_c$ all that is necessary is that the K shell be unfilled (the remaining shells of the atom may remain filled). In principle, such a situation arises after the Auger effect in π^- or μ^- -mesic atoms.