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CRITICAL CHARGE IN COLLISIONS OF NUCLEI

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The critical charge Z_c is defined as that value of the nuclear charge at which the energy of the ground level of the electron reaches the limit of the lower continuum $\varepsilon = -1$ ($\hbar = c = m_e = 1$). At $Z > Z_c$ the Coulomb field of the bare nucleus[1)] produces two electron-positron pairs, the electrons of which settle on the 1S level, and the positrons go off to infinity through the Coulomb barrier (a detailed discussion of the properties of the stable system produced - the supercritical atom - produced after the positron emission can be found in [1]). According to calculations [2], $Z_c = 170$ for the isolated nucleus. Therefore, in spite of the latest progress in searches for superheavy elements [3, 4], the possibility of the existence of nuclei with $Z > Z_c$ appears at present to be purely hypothetical.

Apparently a more realistic method of verifying the theory of supercritical atoms [1, 5] is to observe spontaneous quasistatic production of positrons upon collision of heavy nuclei, say two bare uranium nuclei. The idea of such an experiment is that when two nuclei come close together to a distance $R < \hbar/m_e c = 1$ the electron is acted upon by a field analogous to the Coulomb field of a nucleus with double the charge 2A. Therefore, for example, in the limiting case $R = 0$ (coalescence of the nuclei) the critical charge $Z_c(R)$ is decreased by one-half, $Z_c(0) = (1/2)Z_c(\infty) = 85$. For quantitative predictions it is necessary to calculate the dependence of Z_c on the distance between the nuclei R. In view of the large mass of the nuclei, they can be regarded as being at rest (all the more since the probability of positron production increases strongly with decreasing R, and at the instant of closest approach of the nuclei it vanishes). As shown in [6], to solve this problem one can use a variational method. We present below preliminary results of such a calculation.

The motion of the levels with increasing Z, the dependence of Z_c on the radius of the nucleus, and other characteristic features of the relativistic

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[1)] I.e., of a nucleus with all its electrons removed. Incidentally, for the production of positrons at $Z > Z_c$ all that is necessary is that the K shell be unfilled (the remaining shells of the atom may remain filled). In principle, such a situation arises after the Auger effect in π- or μ-mesic atoms.

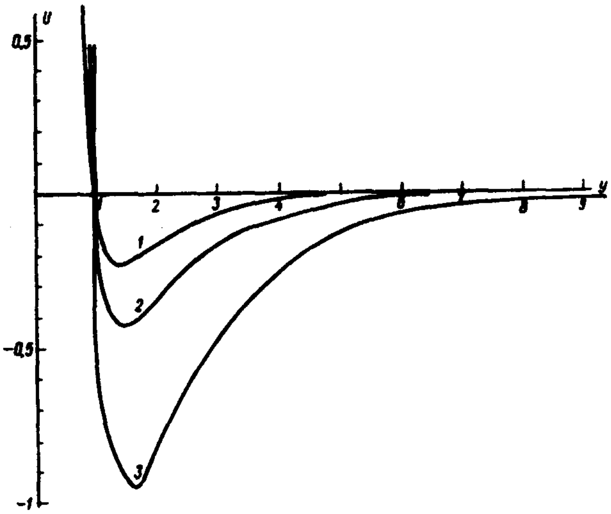

Fig. 1

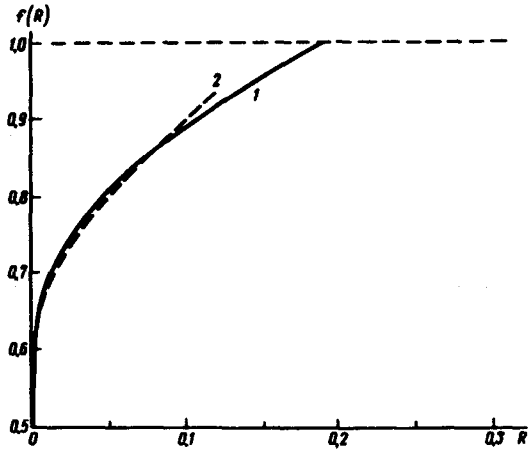

Fig. 2

Fig. 1. Effective potential in Eq. (3). Curves 1 - 3 correspond to α = 0.67 (the uranium nucleus) and R = 1, 0.75, and 0.5 (in units of $\hbar/m_e c$). The abscissas represent the quantity $y = 2\alpha x/R$.

Fig. 2. Decrease of critical charge Z_c when the nuclei come closer together. Curve 1 - results of numerical calculation, curve 2 - calculation from Eq. (7).

Coulomb problem are the same for both spinor and scalar particles [2, 5]. This allows us to start with the case of spin s = 0 (the Klein-Gordon equation with vector coupling), which is simpler from the computational point of view. For simplicity, we consider the symmetrical problem of two centers: $Z_1 = Z_2 = Z$. In the spheroidal coordinates $\xi = (r_1 + r_2)/R$ and $\eta = (r_1 - r_2)/R$ we have the equation [6]:

$$\frac{d}{d\xi}\left[(\xi^2 - 1)\frac{d\psi}{d\xi}\right] = 2\alpha\xi\left(R - \alpha \ln\frac{\xi + 1}{\xi - 1}\right)\psi \ . \tag{1}$$

Here $\alpha = Z/137$ (Z is the charge of each of the nuclei), and R is the distance between them (in units $h/m_e c = 1$). The critical charge $Z = Z_c$ corresponds to the point $\alpha = \alpha_c$ at which Eq. (1) first has a solution that decreases at infinity:

$$\psi \sim \xi^{-3/4}\exp(-\sqrt{8\alpha R\xi}), \quad \xi = \frac{2r}{R} \to \infty \ . \tag{2}$$

The change of variable $\xi = \coth x$ recasts (1) in the form that coincides with the Schrodinger equation for a level with zero binding energy:

$$\psi'' - 2U(x)\psi = 0,$$
$$V(x) = \alpha\frac{\mathrm{ch}\, x}{\mathrm{sh}^3 x}(R - 2\alpha x); \quad 0 < x < \infty \tag{3}$$

(the form of the effective potential U(x) is shown in Fig. 1). This equation was solved numerically; the results are shown in Fig. 2.

We put

$$\frac{Z_c(R)}{Z_c(\infty)} = f(R) \quad . \tag{4}$$

The function f(R) shows the extent to which the critical charge is decreased when the nuclei are brought together to a distance R (it is obvious that $f(\infty) = 1$ and $f(0) = 1/2$).

We note that Eq. (1) was obtained in [6] by a variational method (for a class of functions that depend on ξ but not on η, corresponding to averaging the potential over the variable η). Its solution (curve 1) therefore yields the upper limit for f(R). To verify the accuracy of this approximation, we obtained in analytic form the asymptote for f(R) as $R \to 0$. The wave function near the nuclei is

$$\psi(\xi, \eta) \sim (\xi^2 - \eta^2)^{\sigma/2}, \qquad \sigma = 1 - \sqrt{1 - 4\alpha^2} \tag{5}$$

(see [6]). On the other hand, in the region r_1, $r_2 >> R$ the nuclei can be regarded as a unit, which yields (at $\varepsilon = -1$)

$$\psi = r^{-1/2} K_{i\nu}(\sqrt{8\alpha r}) \sim r^{-1/2} \sin(g\Lambda - g \ln \xi) \;, \tag{6}$$

where $\nu = 2g$, $\Lambda = -(\ln \alpha R + 2\gamma)$, and $\gamma = 0.577$ is Euler's constant.

Expressions (5) and (6) join together in the region $R << r << 1$; this leads to the formula

$$R = \frac{0.315}{\alpha_c} \exp\left\{ - \frac{1}{g} \operatorname{arc\,ctg}\left(\frac{1 - \sqrt{3 - 4g^2}}{2g} \right) \right\} \tag{7}$$

$$(g = 2\sqrt{\alpha_c^2 - 1/16}\,, \qquad \alpha_c = Z_c/137$$

which is asymptotically exact at $R \to 0$. Comparison of curves 1 and 2 on Fig. 2 shows that when $R \lesssim 0.1$ the error of the obtained solution is small.

Thus, the function f(R) from (4) increases rapidly in the region of small R and at $R > 0.3$ it is already close to unity[2]. Therefore for a noticeable decrease of Z_c the colliding nuclei should be brought together to a distance $R \sim 0.1$ which is small compared with the Compton wavelength of the electron.

This conclusion remains in force also for particles with spin 1/2 (electrons). We present here only the analog of formula (7)

$$R = \frac{0.16}{\alpha_c} \exp\left\{ - \frac{1}{g} \operatorname{arc\,ctg}\left(- \frac{\sqrt{1 - \alpha_c^2}}{g} \right) \right\}, \quad g = \sqrt{4\alpha_c^2 - 1}\,. \tag{8}$$

A detailed derivation of these formulas, and also the formulation of the variational principle for the critical charge Z_c in the case of particles with spin 1/2 will be published separately.

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[2]) This agrees also with the form of f(R) as $R \to \infty$: $f(R) = 1 - \beta R^{-4}$, where the coefficient β has a small numerical value ($\beta \sim 10^{-2} - 10^{-3}$).

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GENERATION OF WAVES BY A ROTATING BODY

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An axially-symmetrical body rotating inside a resonator cavity is capable of amplifying definite oscillation modes inside the resonator, transferring the rotation energy to these oscillations.

The frequency of the amplified oscillations is not an integer multiple of the angular velocity of the body, and the instantaneous state of the resonator does not depend on the time, so that the phenomenon in question differs from the parametric resonator.

In scattering of a plane wave incident on the rotating body, it is advisable to expand the wave into spherical (or cylindrical) waves with different values of momentum projection on the rotation axis. In the scattering, the waves with (sufficiently large) momentum parallel to the rotation vector become amplified, and all others become attenuated. In the presence of an external reflector with small losses (resonator), the amplification following single scattering may turn into generation. The linear velocity on the surface of the rotating body obviously is smaller than the speed of light, $v = \beta c$, $\beta < 1$. The amplified waves have an angular dependence $\exp(in\ \phi)$, where $n > \beta^{-1}$. It follows therefore that the radius of the body is smaller than n wavelengths by at least a factor of β; this means that the body is inside the zone in which the wave amplitude decreases more rapidly than $(r/\lambda)^n$. Therefore at small β the gain is exponentially small, like $\exp(\beta^{-1})$ or even weaker.

The foregoing pertains to a body made of a material that absorbs waves when at rest; the conditions for amplification and generation are obtained after transforming the equations to the moving system. A similar situation can apparently arise also when considering a rotating body in the state of gravitational relativistic collapse.

The metric near such a body is described by the well-known Kerr solution. The gravitational capture of the particles and the waves by the so-called trapping surface replaces absorption; the trapping surface ("the horizon of events") is located inside the surface $g_{00} = 0$. Finally, in a quantum analysis of the wave field one should expect spontaneous radiation of energy and momentum by the rotating body. The effect, however, is negligibly small, less than $\hbar\omega^4/c^3$ for power and $\hbar\omega^3/c^3$ for the decelerating moment of the force (for a rest mass $m = 0$, in addition, we have omitted the dimensionless function β).