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#### GENERATION OF WAVES BY A ROTATING BODY

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An axially-symmetrical body rotating inside a resonator cavity is capable of amplifying definite oscillation modes inside the resonator, transferring the rotation energy to these oscillations.

The frequency of the amplified oscillations is not an integer multiple of the angular velocity of the body, and the instantaneous state of the resonator does not depend on the time, so that the phenomenon in question differs from the parametric resonator.

In scattering of a plane wave incident on the rotating body, it is advisable to expand the wave into spherical (or cylindrical) waves with different values of momentum projection on the rotation axis. In the scattering, the waves with (sufficiently large) momentum parallel to the rotation vector become amplified, and all others become attenuated. In the presence of an external reflector with small losses (resonator), the amplification following single scattering may turn into generation. The linear velocity on the surface of the rotating body obviously is smaller than the speed of light,  $v = \beta c$ ,  $\beta < 1$ . The amplified waves have an angular dependence  $\exp(in\phi)$ , where  $n > \beta^{-1}$ . It follows therefore that the radius of the body is smaller than  $n$  wavelengths by at least a factor of  $\beta$ ; this means that the body is inside the zone in which the wave amplitude decreases more rapidly than  $(r/\lambda)^n$ . Therefore at small  $\beta$  the gain is exponentially small, like  $\exp(\beta^{-1})$  or even weaker.

The foregoing pertains to a body made of a material that absorbs waves when at rest; the conditions for amplification and generation are obtained after transforming the equations to the moving system. A similar situation can apparently arise also when considering a rotating body in the state of gravitational relativistic collapse.

The metric near such a body is described by the well-known Kerr solution. The gravitational capture of the particles and the waves by the so-called trapping surface replaces absorption; the trapping surface ("the horizon of events") is located inside the surface  $g_{00} = 0$ . Finally, in a quantum analysis of the wave field one should expect spontaneous radiation of energy and momentum by the rotating body. The effect, however, is negligibly small, less than  $\hbar\omega^4/c^3$  for power and  $\hbar\omega^3/c^3$  for the decelerating moment of the force (for a rest mass  $m = 0$ , in addition, we have omitted the dimensionless function  $\beta$ ).

The rotating body is regarded classically, and none of the foregoing applies to particles with quantized angular momentum.

Further generalization to the case of fermions, including charged ones, is also possible. The rotating body produces spontaneous pair production in the case when the body can absorb one of the particles, while the other (anti)particle goes off to infinity and carries away energy and angular momentum. All that is necessary is that the momentum carried away be sufficient to draw energy away from the body, which requires a definite value of the impact parameter  $b$ ; the region between the surface of the body and a cylinder of radius  $b$  is the barrier. Finally, there is a possible variant in which particle absorption is replaced by scattering of the particle by the material of the body. Obviously, rotation alone will not lead to pair production without interaction with the body.

To prove all the statements, let us consider the simplest case of a scalar field  $\psi$ . In vacuum  $\psi$  satisfies the equation  $\square\psi - m^2\psi = 0$ . In an absorbing medium in a coordinate system where the medium is at rest we have  $\square\psi + a(\partial\psi/\partial t) - m^2\psi = 0$ , where  $a$  characterizes the damping. In a system where the medium moves along the  $X$  axis the Lorentz transformation yields

$$a \frac{\partial \psi}{\partial t} \rightarrow a \gamma \left( \frac{\partial \psi}{\partial t} - \beta \frac{\partial \psi}{\partial x} \right); \gamma = (1 - \beta^2)^{-1/2}.$$

Let us consider the cylindrical problem  $\psi = f(r) \exp[-i\omega t + in\phi]$ .

Let  $X = r\phi$  be reckoned along the circle over which the rotation takes place,  $\beta = r\Omega/c$ , where  $\Omega$  is the angular velocity of the body.

In this case the additional term in the wave equation inside the rotating body (under its surface) is equal to

$$\psi a \gamma (i\omega + \frac{i\beta n c}{r} (\psi = \psi i a \gamma (\omega + n\Omega))$$

Consequently, the additional term reverses sign at  $n\Omega < -\omega$ , where  $n < 0$  and  $|n| > \omega/\Omega$ . The medium operates effectively as an amplifier and not as an absorber with respect to waves with such values of  $n$ .

For an  $n$ -pole, the boundary of the wave zone corresponds in order of magnitude to  $r_\omega = |n|\lambda = |n|c/\omega$ ; the radius of the body is  $r = v/\Omega = \beta c/\Omega$ . From the inequality needed for amplification we get  $r/r_\omega < \beta$ , the body lies deep inside  $r_\omega$ . The condition for wave amplification coincides with the following simple energy criterion: the photon energy is  $E = \hbar\omega$ , the angular momentum of the photon in the  $n$ -pole state is  $\mu = n\hbar$  (we neglect the spin  $\pm 1$ ), and  $\mu\Omega > E$  denotes that the decrease in the body's rotation energy is larger than the energy of the emitted photon. The constant  $\hbar$  does not enter in the solution, being a gift to the modern method of expression in an era when "quantum mechanics helps understand classical mechanics."

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