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SIMPLE HIGH-ENERGY SCATTERING MODEL

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Attempts have been made recently to consider high-energy scattering in the framework of SU(6) [1] or SU(3) symmetry together with some additional assumptions concerning the character of the interaction process [2] (in [2] the small-angle scattering of particles is connected only with single quark-quark and quark-antiquark collisions). The relations between the cross sections of the different reactions, obtained in these papers, agree well with experiment. It is of interest to consider other variants of the interaction process at high energies and to compare them with the experimental data. We consider below a model in which elastic forward scattering is only via single quark-antiquark or three-quark scattering. This model is similar to the model of Levin and Frankfurt [2], but it is assumed here that the only essential quark interactions are those leading to bound states (mesons and baryons), and the remaining interactions are small. Further, we shall assume, as in [1], that SU(6) symmetry obtains. It was noted in [3] that the relations that follow from SU(6) symmetry are not satisfied in nucleon-antinucleon annihilation into two mesons, but it is perfectly possible that SU(6) symmetry is valid not for large t (annihilation process), but in the interval $t \sim 4\mu^2$.

We must also note the following circumstance: It follows from the experimental data that at energies larger than 15 BeV the different SU(6)-invariant amplitudes for quark-quark scattering are equal to one another in this model. This means that the SU(6) symmetry is in essence not employed at such energies.

Under the foregoing assumptions, the meson-baryon forward-scattering amplitudes are expressed in terms of two SU(6)-invariant quark-antiquark scattering amplitudes (synplet and 35-plet) and in terms of two three-quark scattering amplitudes (56-plet and 70-plet). This leads to the following relations between the cross sections

$$\sigma_{\pi^+p} - \sigma_{\pi^-p} = \sigma_{K^+n} - \sigma_{K^-n} = (1/2)(\sigma_{K^+p} - \sigma_{K^-p}), \tag{1}$$

$$\sigma_{K^-p} + \sigma_{K^+p} = \sigma_{K^-n} + \sigma_{\pi^+p} . \tag{2}$$

Relation (1) is the Johnson-Treiman equation [1], while (2) is that of Levin and Frankfurt [2]. Relation (2) at energies 6 - 20 BeV is satisfied with ~15% accuracy. It is natural to assume that to obtain more exact relations between the cross sections it is necessary to take into account the breaking of SU(6) symmetry. It can be assumed, for example, that at high energies the SU(6)-invariance is violated only in interactions where a third quark participates. In these cases the quark-antiquark and three-quark scattering amplitudes a_μ and b_ρ (μ and ρ number the multiplets) must be replaced by $a_\mu + \alpha_\mu$ and $b_\rho + \beta_\rho$, where α and β are additions which break the SU(6) invariance. This leaves only one relation

$$\sigma_{\pi^- p} - \sigma_{\pi^+ p} = \sigma_{K^- p} - \sigma_{K^- n} + \sigma_{K^+ n} - \sigma_{K^+ p}, \qquad (3)$$

which is well satisfied.

In this model, both the baryon-baryon and meson-baryon scattering is due to only single scattering of the quarks making up these particles, and therefore the amplitudes of these processes may be related. It is apparently reasonable to write such relations only at very high energies (above 15 BeV), when the total cross sections are approximately constant, so that the question of the energies at which they are to be compared does not arise. At such energies we have $\sigma_{\pi^- p} \simeq \sigma_{\pi^+ p}$, $\sigma_{K^- p} \simeq \sigma_{K^- n}$, $\sigma_{K^+ p} \simeq \sigma_{K^+ n}$, $\sigma_{pn} \simeq \sigma_{pp}$, and $\sigma_{\bar{p}p} \simeq \sigma_{\bar{p}n}$, from which it follows that $a_1 = a_{35}$ and $b_{70} = b_{56}$. As already mentioned, this means in fact that SU(6) symmetry is not employed at these energies. The quark-antiquark and three-quark scattering amplitudes are then

$$a_{ik} = a + (\delta_{i3} + \delta_{k3})\alpha, \qquad b_{ik\ell} = b + (\delta_{i3} + \delta_{k3} + \delta_{\ell 3})\beta. \qquad (4)$$

Using (4), we can readily obtain the following relations between the cross sections

$$\sigma_{KK} = \sigma_{K\bar{K}}, \quad \sigma_{\pi K} = \sigma_{\pi\bar{K}}, \qquad (5a)$$

$$\sigma_{\Sigma N} - \sigma_{NN} = \sigma_{\Xi N} - \sigma_{\Sigma N} = 3(\sigma_{N\bar{K}} - \sigma_{N\pi}), \qquad (5b)$$

$$\sigma_{N\bar{\Xi}} - \sigma_{N\bar{\Sigma}} = \sigma_{N\bar{\Sigma}} - \sigma_{N\bar{N}} = \sigma_{KN} - \sigma_{\pi N} = 3(\sigma_{K\pi} - \sigma_{\pi\pi}) = 3(\sigma_{KK} - \sigma_{\pi\pi}), \qquad (5c)$$

$$\sigma_{\pi\pi} = \frac{2}{9}\sigma_{N\bar{N}}, \qquad (5d)$$

$$\sigma_{\pi N} = \frac{1}{3}\sigma_{N\bar{N}} + \frac{1}{6}\sigma_{NN}. \qquad (5e)$$

Of all these relations, only (5e) can be checked at present. It is in good agreement: at 20 BeV we have $\sigma_{\pi N} = 24$ mb, $\sigma_{N\bar{N}} = 50$ mb, and $\sigma_{NN} = 39$ mb [4]. It is curious to note that the predicted numerical value of $\sigma_{\pi\pi}$ in this model is very close to the value of $\sigma_{\pi\pi}$ in the factorizable theory of complex momenta ($\frac{2}{9}\sigma_{N\bar{N}} \simeq 11$ mb, $\sigma^2_{\pi N}/\sigma_{NN} \simeq 13$ mb). The values of $\sigma_{\Sigma N}$ and $\sigma_{\Xi N}$ predicted by the formulas in (5) are relatively small: $\sigma_{\Sigma N} \simeq 28$ mb and $\sigma_{\Xi N} \simeq 16$ mb.

A characteristic feature of the model is that relations are obtained between $\sigma_{\pi N}$, σ_{NN}, $\sigma_{N\bar{N}}$, and $\sigma_{\pi\pi}$. However, the fulfillment of relation (5e) can apparently be regarded only as evidence in favor of the composite model of particles (the baryons consist of more quarks than the mesons), for any choice of quark interaction (in the Levin-Frankfurt model) leads to a relation between σ_{NN}, $\sigma_{N\bar{N}}$, and $\sigma_{\pi N}$ ($\sigma_{\pi N} = \frac{1}{3}\sigma_{NN} + \frac{1}{3}\sigma_{N\bar{N}}$) which is qualitatively satisfied, and the predicted value of $\sigma_{\pi\pi}$ ($\sigma_{\pi\pi} = \frac{2}{3}\sigma_{\pi N} \simeq 16$ mb) is numerically close to that given by (5e).

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CRITICAL TEMPERATURE OF THIN SUPERCONDUCTING FILMS

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1. Interest in inhomogeneous superconducting systems has noticeably revived recently in connection with the publication of experimental data [1] offering evidence that the critical temperature of thin films increases with decreasing film thickness, and confirming apparently the existence of the surface superconductivity effect [2]. Calculations for inhomogeneous superconducting systems are greatly hampered by the nonlinearity of the superconductivity-theory equations. Most available results pertain therefore to the case of weakly inhomogeneous superconducting systems, the scale of inhomogeneity in which is large compared with the coherence length ξ_0 or with the mean free path l.

Of definite interest, especially from the point of view of the aforementioned experiments, is the opposite case, that of superconductors with small inhomogeneity scale. It turns out that such systems lend themselves to calculations if they are bounded in those directions in which they are inhomogeneous; more accurately, if the corresponding dimensions are small compared with ξ_0 and l. We consider in this note a typical example of "pure" ($l \gg \xi_0$) film with an interaction parameter $\lambda(z)$ which is variable over the thickness. We are interested essentially in the critical temperature of the film as a whole, writing it in the usual form

$$T_c \sim \omega_0 \exp[-2\pi^2/\lambda^* \, m \, p_0], \tag{1}$$

with effective interaction parameter λ^* that depends on the film thickness d and on the form of the function $\lambda(z)$.

2. The critical temperature and the "energy gap" $\Delta^*(z) = \lambda(z)\tilde{F}^+(\vec{x}, t, \vec{x}, t)$ 1) near T_c are determined, within the framework of the Gor'kov scheme [3,4], by the following equation

$$\Delta^*(z) = \int dz' \, K(z, z') \, \Delta^*(z') \, , \tag{2}$$

where

$$K(z,z') = \frac{m}{2\pi} \sum_{mn} K_{mn} \, \psi_m^*(z) \, \psi_n^*(z') \, \psi_n(z) \, \psi_m(z') \, \lambda(z) \, ,$$

$$K_{mn} = T_c \sum_{\omega} \int d\zeta \, \frac{f(\zeta, E_m) f(\zeta, E_n)}{(i\omega - \zeta - E_m)(i\omega + \zeta + E_n)} \, .$$