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CRITICAL TEMPERATURE OF THIN SUPERCONDUCTING FILMS

D. A. Kirzhnits and E. G. Maksimov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
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1. Interest in inhomogeneous superconducting systems has noticeably revived recently in connection with the publication of experimental data [1] offering evidence that the critical temperature of thin films increases with decreasing film thickness, and confirming apparently the existence of the surface superconductivity effect [2]. Calculations for inhomogeneous superconducting systems are greatly hampered by the nonlinearity of the superconductivity-theory equations. Most available results pertain therefore to the case of weakly inhomogeneous superconducting systems, the scale of inhomogeneity in which is large compared with the coherence length ξ_0 or with the mean free path l.

Of definite interest, especially from the point of view of the aforementioned experiments, is the opposite case, that of superconductors with small inhomogeneity scale. It turns out that such systems lend themselves to calculations if they are bounded in those directions in which they are inhomogeneous; more accurately, if the corresponding dimensions are small compared with ξ_0 and l. We consider in this note a typical example of "pure" ($l \gg \xi_0$) film with an interaction parameter $\lambda(z)$ which is variable over the thickness. We are interested essentially in the critical temperature of the film as a whole, writing it in the usual form

$$T_c \sim \omega_0 \exp[-2\pi^2/\lambda^* m p_0], \tag{1}$$

with effective interaction parameter λ^* that depends on the film thickness d and on the form of the function $\lambda(z)$.

2. The critical temperature and the "energy gap" $\Delta^*(z) = \lambda(z)\tilde{F}^+(\vec{x}, t, \vec{x}, t)$ 1) near T_c are determined, within the framework of the Gor'kov scheme [3,4], by the following equation

$$\Delta^*(z) = \int dz' \, K(z, z') \, \Delta^*(z') \, , \tag{2}$$

where

$$K(z,z') = \frac{m}{2\pi} \sum_{mn} K_{mn} \, \psi_m^*(z) \, \psi_n^*(z') \, \psi_n(z) \, \psi_m(z') \, \lambda(z) \, ,$$

$$K_{mn} = T_c \sum_\omega \int d\zeta \, \frac{f(\zeta, E_m) f(\zeta, E_n)}{(i\omega - \zeta - E_m)(i\omega + \zeta + E_n)} \quad .$$

The latter quantity is equal to $\ln(\omega_0/T_c)$ when $|E_m - E_n| \lesssim T_c$, to $\ln[2\omega_0/|E_m - E_n| - 1]$ when $|E_m - E_n| \gg T_c$, and to zero when $|E_m - E_n| > 2\omega_0$. Owing to the presence of the factor f, the summation over the levels is carried out here and below within the Fermi boundary.

In the weak-coupling limit (which corresponds to neglecting the possible pre-exponential factor in (1)), only the region $|E_m - E_n| \lesssim T_c$ is of importance. The number of diagonal K terms in this region is $\sim p_0 d$, and that of the off-diagonal terms is at the most $\sim md^2T_c$. Therefore, the off-diagonal terms can be left out of the kernel of (2) if the condition

$$d \ll \xi_0 = p_0/mT_c \tag{3}$$

is satisfied. We note that the weak-coupling assumption becomes unnecessary if the more stringent condition $d \ll p_0/m\omega_0$ is satisfied [2].

Assuming the condition (3) to be satisfied, we retain in $K(z, z')$ only the diagonal terms (there are no off-diagonal terms also in the opposite case of homogeneous superconducting systems). Introducing the notation

$$\int dz\, \Delta^*(z)\,|\psi_m(z)|^2 = \Delta^*_m\,, \qquad \int dz\, \lambda(z)\,|\psi_m(z)|^2 |\psi_n(z)|^2 = \lambda_{mn}\,,$$

we obtain

$$\Delta^*(z) = \frac{\pi}{p_0\lambda^*}\,\lambda(z) \sum_m \Delta^*_m |\psi_m(z)|^2, \qquad \Delta^*_m = \frac{\pi}{p_0\lambda^*} \sum_n \lambda_{mn}\,\Delta^*_n\,. \tag{4}$$

The critical temperature is determined by the eigenvalue of the last equation, i.e.,

$$\det\left|\delta_{mn} - \frac{\pi}{p_0\lambda^*}\,\lambda_{mn}\right| = 0\,. \tag{5}$$

This equation can have more than one solution, a fact which we shall discuss elsewhere. We consider here the maximum value of T_c.

3. From the point of view of the superconductivity surfaces (more accurately, the surface amplification of superconductivity [2,5]), it is of interest to carry out the calculations for a film with $\lambda(z) = \lambda + \lambda_s$ having boundary regions of thickness $d_s/2 \ll d$ each and a central region $\lambda(z) = \lambda$ thick. We assume for simplicity that the electrons are in a one-dimensional square-well potential of width d and depth $U > \mu$. Using the corresponding expressions for ψ_m and E_m and recognizing that in practice $p_0 d_s \gg 1$, we obtain after simple calculations

$$\Delta^*(z) \sim \lambda(z) \sum_m |\psi_m(z)|^2\,, \tag{6}$$

$$\lambda^* = \frac{1}{d}\int dz\, \lambda(z) = \lambda + \lambda_s d_s/d\,. \tag{7}$$

If the terms $\sim 1/p_0 d_s$ must be taken into account, it is necessary to introduce in the second term of (7) a factor [3] $1 + \pi\varphi/4p_0 d_s$, where φ is a slowly-varying function of μ/U ($\varphi(0) = 1$, $\varphi(1) = 47/15$). We present also an expression for λ^* when $p_0 d_s < 1$

$$\lambda^* = \lambda + \frac{4}{5}\,\lambda_s\,\frac{d_s}{d}\left(\frac{\mu}{U}\right)^2\,. \tag{8}$$

A common feature of the obtained expressions is the linear-fraction character of the variation of ln T_c with the film thickness d [4)]; a similar variation is observed in the experimental data for aluminum films with oxide coatings [1]. For a more detailed comparison with these data we need reliable estimates of the thickness of the surface layer d_s and the mean free path ℓ.

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1) The tilde denotes that it is necessary to introduce in the Fourier expansion in x and y and in the expansion in the transverse-motion functions $\psi_m(z)$ with energy E_m a factor $f(\zeta, E_m) = \theta(\omega_0 - |\zeta + E_m|)$, where $\zeta = [(p_x^2 + p_y^2)/2m] - \mu$.

2) However, if the condition $md^2T_c \lesssim 1$ is satisfied, it becomes easy to consider also the case of arbitrary temperatures.

3) The dependence of this factor, and also of the second term in (8), on U is a reflection of the inhomogeneity of the particle distributions transverse to the film.

4) We note that such a dependence (in the simplest case when $p_0 d_s \gg 1$ and $\lambda = 0$) was obtained from qualitative considerations by Cooper [6] (see also [4]).

CP-ODD WEAK INTERACTION

E. P. Shabalin
Institute of Theoretical and Experimental Physics
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Two different effects - CP-parity nonconservation in $K \to 2\pi$ decays [1] and the absence of weak interaction of neutral currents [2,3] - can be simply explained if the weak-interaction Lagrangian has negative CP-parity

$$L_{qr} = iG/\sqrt{2}\left[j_\alpha^{(q)}\ j_\alpha^{(r)+} - j_\alpha^{(q)+}\ j_\alpha^{(r)}\right], \quad (1)$$

where the indices q and r number the individual terms of the sequence of different weak currents