CONCERNING THE STABILITY OF PHASED OSCILLATORS

J. Rowlands (UKAEA, Culham Lab., Culham, Abingdon, Berks.), V. B. Krasovitskii, and V. I. Kurilko
Physico-technical Institute, Ukrainian Academy of Sciences
Submitted 19 October 1965
JETP Pis'ma 2, 511-514 (1 December 1965)

It is well known that a beam of oscillators made up of plasma particles rotating around a magnetic field is unstable against excitation of oscillations if the distribution function does not depend on the phase in velocity space (in the case when the oscillators are not in phase). It is of interest to investigate the stability of a system of phased oscillators, i.e., oscillators with fixed phase in velocity space. Such a system can be obtained, for example, when a transverse electromagnetic wave propagates in a plasma along a magnetic field. In this case the problem of stability of a system of phased oscillators is identical with the problem of stability of a wave propagating in a plasma along the magnetic field.

We shall consider this problem in the hydrodynamic approximation. The initial system of equations consists of the hydrodynamic equations for the plasma particles and Maxwell's equations:

$$\begin{aligned} &m\frac{d\vec{v}}{dt} = e\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{H}\right), \\ &\mathrm{curl}\,\vec{H} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi}{c}en\vec{v}, \\ &\mathrm{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial\vec{\mathcal{H}}}{\partial t}, \\ &\frac{\partial n}{\partial t} + \mathrm{div}(n\vec{v}) = 0 . \end{aligned} \tag{1}$$

We seek solutions of this system in the form

$$\begin{aligned} &v_x + iv_y = a(t)\exp[-i\Phi + i\varphi(t)], \\ &E_x + iE_y = i\mathcal{E}(t)\exp[-i\Phi + i\psi(t)], \\ &H_y - iH_x = i\mathcal{H}(t)\exp[-i\Phi + i\kappa(t)], \\ &v_z = v_z(t),\ E_z = E_z(t),\ H_z = H_0 = \mathrm{const},\ \Phi \equiv kz - \omega t. \end{aligned} \tag{2}$$

In the equilibrium state we have $v_{z0} = E_{z0} = 0$ and $\kappa_0 = \varphi_0 = \psi_0 = \mathrm{const}$. The amplitudes of the velocities and of the fields are then constant and related by

$$\begin{aligned} &\mathcal{E}_0 = \frac{\omega}{kc}\mathcal{H}_0, \\ &\mathcal{H}_0 = \frac{\omega}{kc}\mathcal{E}_0 + \frac{4\pi}{kc}en_0a_0, \\ &ma_0(\omega_H - \omega) = e\mathcal{E}_0, \end{aligned} \tag{3a}$$

and the wave number k is determined by the dispersion equation

$$k^2c^2 = \omega^2 + \omega_p^2 \frac{\omega}{\omega_H - \omega} . \tag{3b}$$

Let us consider small perturbation of the stationary state, described by formulas (3)

$$\mathcal{E}(t) = \mathcal{E}_0 + \mathcal{E}_1(t), \quad \mathcal{H}(t) = \mathcal{H}_0 + \mathcal{H}_1(t), \quad a(t) = a_0 + a_1(t),$$

$$\kappa(t) = \kappa_0 + \kappa_1(t), \quad \varphi(t) = \varphi_0 + \varphi_1(t), \quad \psi(t) = \psi_0 + \psi_1(t),$$

and obtain the following equation for λ, assuming that all the perturbations are proportional to $\exp(\lambda t)$:

$$\lambda^4 + \lambda^2\{\mu^2 \omega_H \Delta + 2\omega_p^2 \frac{\omega_H}{\Delta} + 4\omega^2 + \Delta^2\}$$
$$+ \{4\omega^2 \omega_H \Delta \mu^2 + \omega_p^2 \omega_H^2 \mu^2 + 4\omega_H \omega \omega_p^2 + 4\omega^2 \Delta^2 + \omega_p^4 \frac{\omega_H^2}{\Delta^2}\} = 0, \tag{4}$$

where

$$\Delta \equiv \omega_H - \omega, \qquad \Omega_0 \equiv e\mathcal{H}_0/mc, \quad \mu^2 = \frac{\Omega_0^2}{\Delta^2} (1 + \omega^2/\lambda^2)^{-1} .$$

A qualitative investigation of this equation shows that it always has solutions corresponting to instability when the field amplitude is sufficiently large. For a quantitative estimate of the growth increments and of the instability conditions, let us consider the simplest limiting cases.

1. Low plasma density (strong magnetic fields). This case comes closest to the problem of stability of a particle accelerated by an external field.

Two limiting cases are then possible:

a) $$\omega_p \ll |\Delta| \ll \omega_H.$$

In this case

$$\lambda^2 = -\Delta^2 - \omega_H \Delta \mu_0^2, \quad \mu_0^2 \equiv \Omega_0^2/\Delta^2.$$

b) $$\omega_p/\omega_H \sim |\Delta|/\omega_H \sim \Omega_0^2/\omega_H^2 \ll 1,$$

$$\lambda^2 = -\Omega_0^2 \frac{\omega_H}{\Delta} + \frac{\omega_p^2 \omega_H^2}{4\omega^2 - \Omega_0^2 \omega_H/\Delta} \frac{\Omega_0^2}{\Delta^2} . \tag{5}$$

Thus, the instability can exist only in the region of fast waves ($\omega > \omega_H$), and with increasing wave amplitude both the width of the instability region (in terms of frequency) and the instability increment increase. It can be shown that the results of case (a) can be obtained from an analysis of the problem of the stability of motion of one particle in an external field.

According to (b) the instability increment decreases with increasing plasma density in this region.

2. High plasma density (weak magnetic fields). When

$$\omega_H/\omega_p \sim k^2c^2/\omega_p^2 \sim \Omega_0^2/\omega_p^2 \sim (\omega/\omega_p)^{1/2} \ll 1 \tag{6}$$

the solution of (4) is

$$\lambda^2 = -\omega_p^2 \{1 + \delta \pm \sqrt{\delta^2 - \omega_H \Omega_0^2 / \omega_p^3}\}; \quad \delta \equiv \frac{k^2c^2 + \omega_H\omega_p}{\omega_p^2} .$$

The instability has therefore a threshold character in this case ($\Delta > 0$), too [1)].

We can analyze similarly the case of finite wavelength for the perturbation of the longitudinal field and of the velocity. In the latter case the instability conditions and the increments obtained above are altered. In addition, the order of Eq. (4) increases, and this can lead to new instabilities, similar, for example, to those considered in [1].

It must be noted that the problem of stability of curvilinear beams of charged particles, which likewise constitute beams of excited oscillators, was first considered in [2]. Unlike the conditions considered there, in our problem the amplitude of the oscillator is uniquely determined by the wave amplitude. Therefore the instability remains in our case even when the plasma density is zero (as noted above, this corresponds to the instability of a particle moving in an external field), whereas in [2] the increment vanishes at zero beam density.

The authors are grateful to Ya. B. Fainberg for a discussion of the results.

One of the authors (J. R.) is grateful to the USSR State Atomic Energy Commission and to the Director of the Ukrainian Physico-technical Institute for hospitality.

[1] T. F. Volkov, ZhTF 31, 1149 (1961), Soviet Phys. Tech. Phys. 6, 838 (1962).

[2] A. V. Gaponov, Izvestiya vuzov, Radiofizika 2, 450 (1959).

1) In this case the instability has a decay character.

EFFECT OF HYDROSTATIC PRESSURE ON THE ELECTRON EFFECTIVE MASS IN InSb

E. S. Itskevich, V. M. Muzhdaba, V. A. Sukhoparov, and S. S. Shalyt
Institute of High-pressure Physics, USSR Academy of Sciences;
Semiconductor Institute, USSR Academy of Sciences
Submitted 20 October 1965
JETP Pis'ma 2, 514-519 (1 December 1965)

It was established in the experiments of Long [1] and Keyes [2] that the electric resistivity ρ and the Hall coefficient R of indium antimonide increase exponentially with the pressure P in the intrinsic-conductivity region. This result was attributed to the linear increase of the forbidden band ϵ_g of the crystal with increasing pressure (up to 12,000 kg/cm^2), the proportionality coefficient being 1.5×10^{-5} eV-cm^2/kg. A consequence of this increase of ϵ_g is the exponential increase of the concentration of the intrinsic carriers. For a numerical determination of the proportionality coefficient in the dependence of ϵ_g on P from