INERTIAL ECHO AND COHERENT GRAVITATIONAL WAVES

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It is universally recognized that one of the most promising experimental verifications of general relativity theory is the detection of gravitational waves [1].

We consider in this paper new physical phenomena - inertial (or gravitonic) induction and echo - which in our opinion can be used for generation and reception of coherent gravitational waves (CGW) in a narrow band of optical frequencies under laboratory conditions. Numerical estimates show that the proposed experimental scheme on the detection of CGW can be realized with present-day technical means.

From the analogy between a weak gravitational field and the electric field [2] it follows that along with photon induction and echo, which were predicted and observed in [3] and [4] respectively, there should apparently exist also inertial induction and echo - the graviton analogs of these phenomena. Gravitonic induction and echo are guided CGW pulses due to coherent oscillations of a tremendous number of multipole moments $D_{ln}^{(m)}$ of the masses of molecules, ions, or nuclei. By the same token, excitation of gravitonic induction and echo differs from excitation of photonic induction and echo in that the external pulsed action gives rise to quantum superposition states not of electric multipoles $D_{ln}^{(e)}$, but of $D_{ln}^{(m)}$, and the oscillations of $D_{ln}^{(m)}$ cause radiation not of coherent electromagnetic waves (CEW), but of CGW. Both the electromagnetic superradiating state $D_{ln}^{(e)}$ and the gravitonic superradiating state $D_{ln}^{(m)}$ can be produced by laser pumping. This is due to the fact that the electric-charge density distribution ρ is closely connected with the distribution of the mass density μ which carries these charges. Therefore, by producing a superradiating electromagnetic state with a laser pump, we also force by the same token a tremendous number of mass multipoles to precess. It is just this cirmustance, together with the high frequency, which ensures radiation of the observed power of the CGW.

A serious obstacle on the path of realization of a CGW generator is the electromagnetic radiative damping of the superradiating state, which unavoidably accompanies the CGW generation and interrupts, after a short time interval τ, the coherent $D_{ln}^{(m)}$ oscillations, However, both experimental [5] and theoretical [6] investigations of the CEW generation process have shown that the time τ can be lengthened many orders of magnitude by reducing the probability of spontaneous CEW emission via enclosing the radiation in an electromagnetic resonator and retaining the CEW radiation in the resonator. On the other hand, the intensity of CGW emission is not noticeably changed thereby, since the electromagnetic resonator cannot confine the gravitational waves, and the CGW emission will take place just as in free space.

It is best to receive the CGW by means of the same medium as is used for generation, but thoroughly shielded against the action of CWE. The CGW reception process can consist of observing the interaction of the CGW gradient with $D_{2n}^{(m)}$. As a result of this process, the medium enters a superradiating electromagnetic state and the emitted photons can be registered with a photomultiplier. By way of an example, let us consider the generagion of CGW by a system of N_1 quadrupoles $D_{2n}^{(m)}$ which are excited by electromagnetic dipole transitions. The operator $D_{2n}^{(m)}$ is obtained from $D_{2n}^{(e)}$ by replacing ρ with $\sqrt{k}\mu$, where k is the gravitational constant. Using [7] and formula (102.10) of [8], we have obtained for the flux of gravitational energy along the y axis:

$$
\begin{aligned}
&I_y = BF, \quad B = |\langle n|T_{21}(S)|n+1\rangle|^2 \{2^2\, 6^3\, \pi\, [d_2\, S\,(2S-1)]^2\}^{-1},\\
&F \sim l^2\, \lambda^2\, k\, m^2\, [Q^{(m)}]^2\, \omega^6 \sin^2(\omega_e\, \Delta t)\, N^2\, C^{-5};\\
&D_{2n}^{(m)} = m\, Q^{(m)}\, [d_2\, S\,(2S-1)]^{-1}\, T_{2n}(S),\\
&T_{2\pm1}(S) = \mp(3/2)^{1/2}\, d_2\, [S_z\, S_\pm + S_\pm\, S_z],\\
&\omega_e = \tfrac{1}{2}\hbar^{-1}\, g_e\, \beta_e\, H_x \sqrt{(S+n)(S-n+1)},\\
&d_2 = 2(5)^{1/2}\, [(2S+3)(2S+2)(2S+1)2S(2S-1)]^{-1/2}, \quad \lambda = \frac{2\pi c}{\omega},
\end{aligned}
\qquad (1)
$$

where l is the length of the sample, $T_{ln}(S)$ a normalized spherical tensor-operator [9], S the angular momentum operator, m the natural mass unit, $Q^{(m)}$ and $Q^{(e)}$ the quadrupole moments of the masses and of the charge ($Q^{(m)} \sim Q^{(e)}$), ω the circular frequency of the light and of the resonant transition between the nondegenerate energy levels E_n and E_{n+1}, N the particle density, H_x the amplitude of the linearly polarized CEW, $|n\rangle$ the eigenfunctions of the particle-energy operator, c the velocity of light, Δt the duration of the exciting light pulse, z the quantization axis for S, β_e the particle electromagnetic dipole measurement unit, and g_e the number of such units.

The graviton-quadrupole interaction of the CGW, for a receiver located at a distance r from the transmitter, is of the form

$$
\begin{aligned}
&\mathcal{H}_{\Gamma Q} = \sum_{j=1}^{N_2} \mathcal{H}_{\Gamma Q}^{j}, \quad \mathcal{H}_{\Gamma Q}^{j} = \sum_{n=2}^{-2} (-1)^n\, T_{2n}(\Gamma)\, T_{2(-n)}^{j}(S)\, \delta_{2|n|} \cos\omega t,\\
&T_{2\pm2}(\Gamma) = (3/2)^{1/2}\, [d_2 S(2S-1)]^{-1}\, m\, Q^{(m)}\, c^2\, R^{\mu}_{0x0},\\
&R^{\mu}_{0x0} = \omega(kI_y/\pi c^7)^{1/2}
\end{aligned}
\qquad (2)
$$

where R_{0x0} is the Riemann curvature tensor component [10], N_2 the number of particles in the receiver, and $\delta_{\alpha\beta}$ the Kronecker symbol.

The electromagnetic-radiation power of the receiver, due to the CGW, is estimated from the formula

$$I^{(e)} \sim g_e^2 \beta_e^2 \omega^6 m^2 [Q^{(m)}]^2 kI_y(\Delta t)^2 N^2 (c^6\hbar^2)^{-1} B_1 l_1^2 \lambda^2, \quad (3)$$

where $B_1 l_1$ is the receiver volume.

The best estimates are obtained with CGW reception and radiation with the aid of the electron shell of ions. Using the constants (in cgs units)

$$g_e = 10^{-2}, \quad \beta_e = 10^{-18}, \quad \omega = 10^{16}, \quad m = 10^{-27}, \quad Q^{(m)} = 10^{-16},$$

$$l_1 = l = 10^3, \quad N = 10^{23}, \quad \Delta t\omega_e = \pi/2, \quad \Delta t' = 10^{-7}, \quad Bl = 10^3, \quad B_1 l_1 = 10^5,$$

we have

$$I_y \sim 10^{-44} N^2 \sin^2(\Delta t\omega_e) = 10^2 \text{ erg/sec}^2 \sim 10^{13} \text{ gravitons/cm}^2 \text{ sec}; \quad (4)$$

$$I^{(e)} \sim 10^{-48} N^2 I_y(\Delta t')^2 = 10^{-14} \text{ erg/sec} \sim 10^{-3} \text{ photon/sec.} \quad (5)$$

It also follows from our analysis that single-mode powerful lasers can emit a measurable power also in the form of CGW, with

$$W_r \sim W_e \theta \xi \kappa \sim W_e 10^{12} N 10^{-41} \sim 10^{-10} W_e \quad \text{(Watts)}, \quad (6)$$

where W_e is the electromagnetic power delivered by the laser, θ the factor by which the electromagnetic transition probability in the resonator is reduced compared to free space, ξ the Townes factor by which the line is narrowed down in the resonator during emission [6] with account of the peculiarities of the superradiating state [11], κ the ratio of the power radiated by CGW and CEW in free space, and N the number of coherently generating particles under the condition that the coordination number of the superradiating state is N/2 [11].

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CORRECTIONS:

In the article "Inertial Echo and Coherent Gravitational Waves," by U. Kh. Kopvillem and V. R. Nagibarov, vol. 2, No. 12, p. 331 (Russ. p. 532), the 12th line from the bottom should read B. Ya. Zel'dovich and not Ya. B. Zel'dovich.