$\langle E' - E\rangle \geq 2E\theta$. These data point out the important fact that the neutron scattering is quasi-elastic near the phase transition point. It is interesting to note that a direct determination of such a change in the scattered-neutron energy is beyond the capabilities of modern experimental techniques.

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SCATTERING OF POLARIZED NEUTRONS IN MAGNETS NEAR THE PHASE-TRANSITION POINT

S. V. Maleev
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences, Leningrad
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Drabkin et al. [1] investigated the scattering of neutrons in nickel at temperatures close to the Curie temperature. We consider in this note the question of the information that can be derived from experiments of this kind.

Using standard methods (see [2]) we can represent the scattering and polarization cross sections in the form

$$\frac{d\sigma}{d\Omega dE'} = 2Q_1 + [1 - (\vec{e}\cdot\vec{m})^2]Q_2 + 2(\vec{P}_0\cdot\vec{e})(\vec{m}\cdot\vec{e})Q_3,$$

$$\frac{d\sigma}{d\Omega dE'}\vec{P}(\vec{q},\omega) = -2(\vec{e}\cdot\vec{m})\vec{e}\,Q_3 - 2(\vec{e}\cdot\vec{P}_0)\vec{e}\,Q_1 \tag{1}$$

$$- \{\vec{P}_0[1 - (\vec{e}\cdot\vec{m})^2] + 2[(\vec{P}_0\cdot\vec{e})(\vec{m}\cdot\vec{e}) - (\vec{P}_0\cdot\vec{m})][\vec{m} - (\vec{e}\cdot\vec{m})\vec{e}]\}Q_2.$$

The quantities Q_i are connected with the atomic-spin correlator as follows:

$$r_0^2\gamma^2\frac{p'}{p}\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}d + \exp[i(\omega t/\hbar)]\sum_{ll'}F_l^*(\vec{q})F_{l'}(\vec{q})\langle S_l^\alpha(t)\exp(-i\vec{q}\cdot\vec{R})_l^{(t)}S_{l'}^\beta(0)\exp(i\vec{q}\cdot\vec{R})_{l'}^{(0)}\rangle$$
$$= Q_1(\vec{q},\omega)\delta_{\alpha\beta} + Q_2(\vec{q},\omega)m_\alpha m_\beta - iQ_3(\vec{q},\omega)\epsilon_{\alpha\beta\gamma}m_\gamma + Q'_{\alpha\beta}(\vec{q},\omega). \tag{2}$$

The notation in (1) and (2) is the same as in [2]; $\omega = E' - E$. Formula (2) is based on symmetry considerations and is valid only for cubic crystals. $Q'_{\alpha\beta}$ has the following property: If $(\vec{a}\cdot\vec{e}) = 0$, the $a_\alpha Q'_{\alpha\beta} a_\beta = 0$. The magnetic-scattering amplitude contains only the atomic-spin components perpendicular to $\vec{e}$, and therefore $Q'_{\alpha\beta}$ does not enter in (1).

Of course, $Q_3 \equiv 0$ in antiferromagnets with two equivalent sublattices. Formulas (1) and (2) are valid only for substances with one or two magnetic sublattices, and in the latter case the atomic spins should be directed along one straight line. The interference between the magnetic and nuclear scattering will not be considered since it is present only in the case of elastic scattering and scattering with absorption and emission of phonons, and is of no importance to us (it is discussed in greater detail in [3,4]).

We can determine Q_3 by investigating the polarization arising when unpolarized neutrons are scattered, and we can determine Q_1 and Q_2 by comparing, say, the cross section for scattering of unpolarized neutrons in two planes, in which $(\vec{e}\cdot\vec{m}) = 0$ and $(\vec{e}\cdot\vec{m}) \neq 0$.

Near the Curie temperature the quantities Q_2 and Q_3 should be small ($Q_{2,3} \equiv 0$ when $T > T_C$) and can be neglected. The scattering is then independent of the magnetization state of the sample.

In investigation of phase transitions we are interested in scattering with low energy transfer and with momentum transfer that differs little from the reciprocal lattice vector $\vec{\tau}$ multiplied by 2π. If $\vec{\tau} \neq 0$, then $\vec{P} \approx -\vec{\tau}(\vec{\tau}\cdot\vec{P}_0)\tau^{-2}$ and the polarization of the scattered protons contains practically no information on the scatterer. However, if $\vec{\tau} = 0$ then the situation changes. Let us direct the x axis along the momentum $\vec{p}$ of the incident neutrons, and the y axis along the scattered-neutron momentum component perpendicular to $\vec{p}$. Then, if $\omega \ll E$ and $\theta \ll 1$, we obtain

$$\vec{P}(\vec{q},\omega) = -\left(\frac{\omega}{2E} P_{0x} + \theta P_{0y}\right)\left(\frac{\omega}{2E}\vec{\epsilon}_x + \theta\vec{\epsilon}_y\right)\left(\frac{\omega^2}{4E^2} + \theta^2\right)^{-1}, \tag{3}$$

where $\epsilon_{x,y}$ are unit vectors in the directions of the coordinate axes.

We are interested in the region of very small ω, in which measurement of the energy dependence of the cross section is practically impossible. The experimentally determined polarization is then connected with $\vec{P}(\vec{q},\omega)$ and the scattering cross section in the following manner

$$\vec{P}(\theta) = \int d\omega\, \vec{P}(\vec{q},\omega)\, \frac{d\sigma}{d\Omega\, dE'} \left(\int d\omega\, \frac{d\sigma}{d\Omega\, dE'}\right)^{-1} \tag{4}$$

Let us consider two cases:

1. $\vec{P}_0 = P_0\vec{\epsilon}_x$. Here:

$$P_x^{(1)}(\theta) = -P_0 Q_1^{-1}(\theta)\int d\omega Q_1(q,\omega)\, \omega^2/4E^2 \left(\frac{\omega^2}{4E^2} + \theta^2\right)^{-1},$$
$$P_y^{(1)}(\theta) = -P_0 Q_1^{-1}(\theta)\theta\int d\omega Q_1(q,\omega)\, \omega/2E \left(\frac{\omega^2}{4E^2} + \theta^2\right)^{-1}; \tag{5}$$

2. $\vec{P}_0 = P_0\vec{\epsilon}_y$. Here:

$$P_x^{(2)}(\theta) = P_y^{(1)}(\theta),$$

$$P_y^{(2)}(\theta) = -\theta^2 P_0 Q_1^{-1}(\theta)\int d\omega\; Q_1(q,\omega)\left(\frac{\omega^2}{4E^2} + \theta^2\right)^{-1}, \qquad (6)$$

$$Q_1(\theta) = \int d\omega\; Q_1(q,\omega).$$

We note further that $P_y^{(2)} + P_x^{(1)} = -P_0$.

Investigation of the quantities $P_x^{(1)}$ and $P_y^{(2)}$ makes it possible to determine the temperatures at which the energy transfer becomes comparable with $2E\theta$, and an investigation of $P_y^{(1)}$ shows the extent to which the cross section with $\omega > 0$ differs from that with $\omega < 0$.

It must be noted that both the scattering considered above and nuclear scattering will contribute to the observed cross section and polarization, as will also the scattering by conduction electrons, which in our case can be quite large. The corresponding terms should, generally speaking, be added to expression (4). However, the two latter types of scattering should not change noticeably in the region of temperatures of interest to us.

A detailed discussion of polarization effects in scattering of this type is found in a paper by Ginzburg and the author [5].

Let us make one more remarks. The thermodynamic potential of a magnet can be represented in the form $\Phi(T,M) = \Phi_0(T) + \Phi_1(T,M)$, where $\Phi_1(T,0) \equiv 0$. Here M is the spontaneous magnetization of the sublattice.

Assuming only simple exchange interaction between the atomic spins, we have:

$$\Phi_0(T) \sim \int d\omega\; d\vec{q}\; J(\vec{q})\; p/p'\; |F(\vec{q})|^{-2}[Q_1(\vec{q},\omega) + \tfrac{1}{3}Q''_{\alpha\alpha}(\vec{q},\omega)].$$

Here $J(\vec{q})$ is the Fourier component of the exchange integral, and $Q''_{\alpha\alpha}(\vec{q},\omega) = Q'_{\alpha\alpha}(\vec{q},\omega)$ when $M = 0$. Apparently it follows from experiment [1] that $d\sigma/d\Omega = 2Q_1(\theta)$ is singular when $T \to T_C$.

It this is the case, then we should expect the thermodynamic potential $\Phi_0(T)$ to be likewise singular when $T \to T_C$, provided the singularity in Q_1 is not cancelled by the singularity which enters in $Q''_{\alpha\alpha}$.

In conclusion, the author thanks G. M. Drabkin for a preprint of [1].

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