lated by reckoning from the most intense mode.

DISPERSION OF SOUND IN SUPERFLUID HELIUM

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At low frequencies ($\omega\tau \ll 1$, ω = frequency of sound, τ = some characteristic times) the absorption of sound is investigated with the aid of the hydrodynamic equations [1]. The question of sound propagation at not too low frequencies ($\omega\tau \gtrsim 1$) is considered with the aid of the kinetic equations. These, together with the equations for the continuity of the mass and of the superfluid liquid, constitute a complete system of equations describing the propagation of sound in helium II.

This question is dealt with in [2]. The favorable situation with respect to establishment of energy equilibrium of the excitations in the gas leads essentially to an exact solution of the problem. The point is that the effective cross section for roton-roton scattering is sufficiently large so that in practice local equilibrium always exists in the roton gas, i.e., the roton gas can be described by a quasi-equilibrium distribution function with a temperature T_r varying from point to point and with a roton-gas relative velocity $\vec{v}_{nr} - \vec{v}_s$. The phonon-phonon scattering is not accompanied by a change in the directions of the colliding-phonon momenta and is anomalously large for small-angle phonon collisions. This process is faster than all other processes that take place with phonons, and always results in an energy equilibrium for phonons moving in a specified direction. In addition, for phonons colliding at small angles when $T < 1.2°K$, the five-phonon process is also faster than the process of phonon scattering by rotons, and there is therefore time for equilibrium with respect to the number of phonons to become established. Thus, the phonons moving in a specified direction can be described by a quasi-equilibrium distribution function that depends at $T < 1.2°K$ on the temperature T_{ph} and the relative velocity $\vec{v}_{nph} - \vec{v}_s$ of the phonons in this direction. When $T \geq 1.2°K$ this distribution function depends also on a certain chemical potential, which is also a function of the phonon direction.

The process with lowest relative speed is the scattering of phonons by rotons. Because of the slowness of this process, establishment of an energy equilibrium between the roton and phonon gases is difficult, although each individual gas is in equilibrium. This circumstance gives rise in the hydrodynamic approximation to some second viscosity, and in the general case to absorption and dispersion of first and second sounds.

The effects of dispersion and absorption of first sound are proportional to the small quantity ρ_{nph}/ρ (ρ_{nph} is the phonon part of the normal density). This makes it possible to separate from the above-mentioned system of equations, describing the propagation of sound oscillations in helium II, two pairs of equations, describing propagation of first and second sound respectively. The condition for the compatibility of each pair of equations is obtained from two dispersion equations that determine the propagation velocities and the absorption coefficients of the corresponding sounds. In the most interesting region of temperatures (below 1.2°K) the theory contains in this case two characteristic times, τ_{phr} and τ_{phph}, which are connected with phonon-roton and phonon-phonon scattering at large angles, and which can be accurately calculated. The region of dispersion of first-sound sets is for frequencies satisfying the conditions $\omega\tau_{phr} \sim 1$ and $\omega\tau_{phph} \sim 1$. To observe dispersion of second sound it is necessary to satisfy the more liberal condition $\omega\tau_{phr} \sim u_2/u_1$ (u_1 and u_2 are the speeds of first and second sound, $u_2/u_1 \lesssim 1$). In the limit when $\omega\tau_{phr} \gg u_2/u_1$ the second sound will propagate, without damping, through the roton gas with velocity $u_{2\infty} = \{(\rho_{sr}/\rho_{nr})(\sigma_r^2/(\partial\sigma_p/\partial T))\}^{1/2}$, which differs strongly from the equilibrium value $u_{20} = \{(\rho_s/\rho)(\sigma^2/(\partial\sigma/\partial T)\rho)\}^{1/2}$ (ρ_n is the density of the normal component, $\sigma = s/\rho$, S is the entropy, and the index r denotes the roton parts of the corresponding quantities).

It is of interest to compare the results obtained in [2] with the latest measurements of the absorption coefficient of second sound by Jeffers and Whitney [3]. According to [2] the first-sound absorption coefficient at temperatures lower than 1.2°K is

$$\alpha_1 = \frac{1}{2}\ \frac{\omega}{c}\ \frac{\rho_{nph}}{\rho}\ \mathrm{Im}(\varphi), \qquad (1)$$

where

$$\varphi = z_{phr} - 3\,\frac{u^2 \ln a + [2uz_{phr} + z^2_{phr}(1-\beta(1-z_{phr})) + 3u^2(1-z_{phr})]}{2+[1-z_{phph}+(1-\beta)(1-z_{phr})]\ln a + 3(1-z_{phph})[1-\beta(1-z_{phr})][-2+(z_{phr}+z_{phph}-1)\ln a]}$$

$$\times \frac{[-2+(z_{phr}+z_{phph}-1)\ln a]}{2+[1-z_{phph}+(1-\beta)(1-z_{phr})]\ln a + 3(1-z_{phph})[1-\beta(1-z_{phr})][-2+(z_{phr}+z_{phph}-1)\ln a]} \qquad (2)$$

$$\left(u = \frac{\rho}{c}\frac{\partial c}{\partial p},\quad \beta = \frac{3kT}{\mu c^2},\quad z_{phr} = 1 - \frac{1}{i\omega\tau_{phr}},\quad z_{phph} = 1 - \frac{1}{i\omega\tau_{phph}},\quad a = \frac{z_{phr}+z_{phph}}{z_{phr}+z_{phph}-2}\right).$$

In (2) we must put $z_{phr} = 1$ at temperatures lower than 0.6°K and $z_{phph} = 1$ at temperatures higher than 0.9°K. In the region of high frequencies ($\omega\tau_{phr}$, $\omega\tau_{phph} \gg 1$) and low ones ($\omega\tau_{phr}$, $\omega\tau_{phph} \ll 1$) it is more convenient to use the limiting expressions for (1) and (2):

$$(\omega\tau_{phr},\ \omega\tau_{phph} \gg 1) \qquad \alpha_1 = \frac{3}{8}\pi\,\frac{\omega}{c}\,(u+1)^2\,\frac{\rho_{nph}}{\rho}\ ; \qquad (3)$$

$$(\omega\tau_{phr},\ \omega\tau_{phph} \ll 1) \quad \begin{cases} \alpha_1 = \dfrac{3}{10}\,\dfrac{\omega^2\tau_{phr}}{c}\,\dfrac{\rho_{nph}}{\rho}\,(u+1)^2 & (T < 0.6°\mathrm{K}), \qquad (4) \\[2ex] \alpha_1 = \dfrac{\omega^2\tau_{phr}}{c}\,\dfrac{\rho_{nph}}{\rho}\left\{\dfrac{2/15}{1+(\tau_{phr}/\tau_{phph})} + \dfrac{(3u+1)^2}{6\beta}\right\} & (T \geq 0.6°\mathrm{K}). \qquad (5) \end{cases}$$

At temperatures above 0.9°K we have $\tau_{phr}/\tau_{phph} \ll 1$ and this term can be neglected in (5).

At high temperatures ($T \geq 1.2$°K) and low frequencies $\omega\tau_{phr} \ll 1$ (this inequality is satisfied in the case of first sound by practically all frequencies that are possible for $T \geq 1.2$°K) we have

$$\alpha_1 = \frac{\omega^2\tilde{\tau}_{phr}}{c}\left\{\frac{2}{15} + \frac{(3u+1)^2}{6\tilde{\beta}}\right\}, \qquad (6)$$

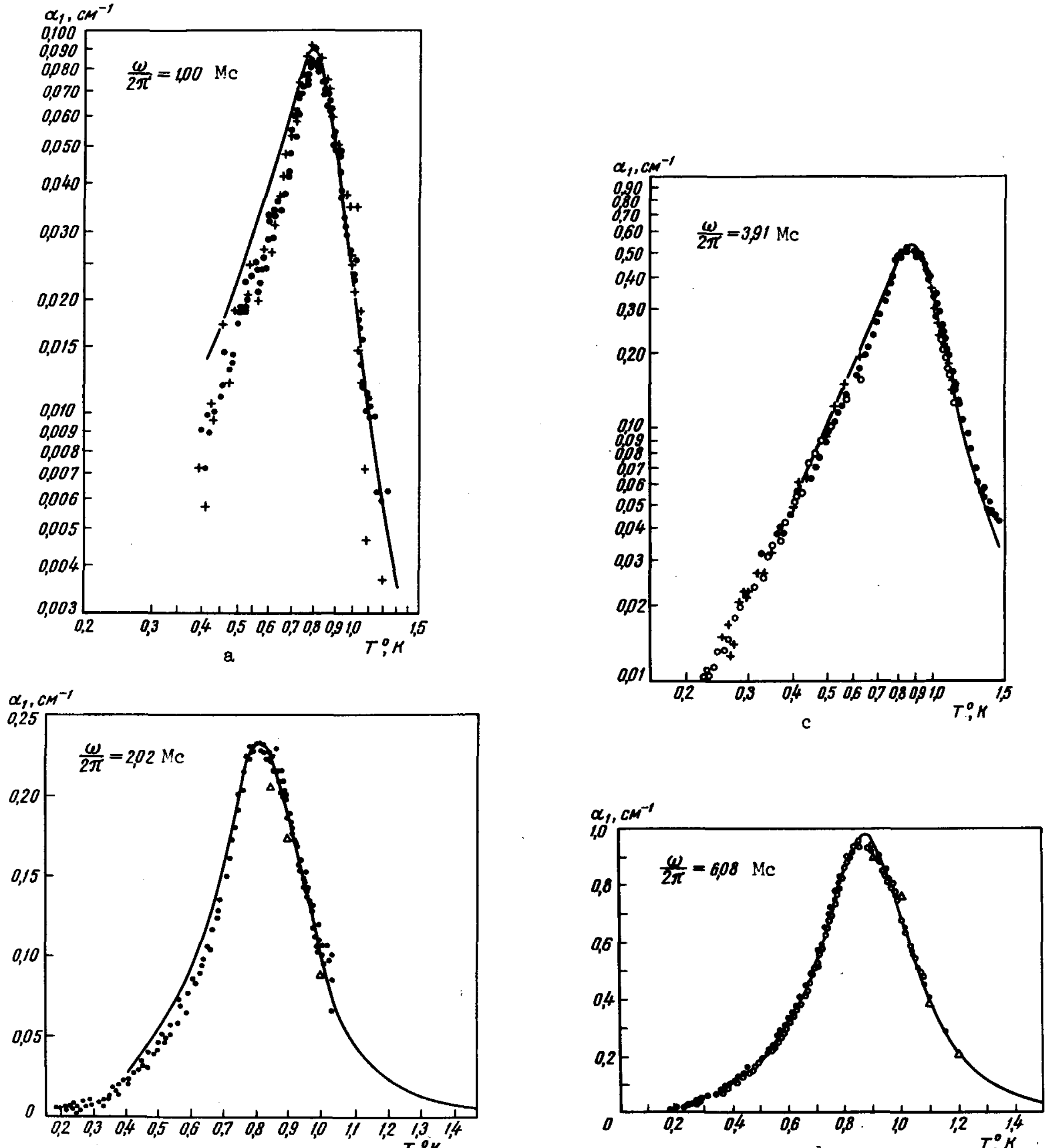
where

$$\tilde{\tau}_{phr} = \tau_{phr}\left[1 + \frac{\frac{1}{8}\left(\frac{\pi^4}{216}-1\right)^2}{\frac{\pi^4}{216}\left(\frac{\tau_{phr}}{\tau_{3\to2}} + \frac{1}{56}\,\frac{\pi^4}{216}\right)}\right]^{-1},$$

$$\frac{1}{\tilde{\beta}} = \left[\frac{1}{\beta} + \frac{\tau_{3\to2}}{\tau_{phr}}\left(\frac{27}{\pi^4} - \frac{1}{9}\right)\right]\left[1 + \frac{\frac{1}{8}\left(\frac{\pi^4}{216}-1\right)^2}{\frac{\pi^4}{216}\left(\frac{\tau_{phr}}{\tau_{3\to2}} + \frac{1}{56} - \frac{\pi^4}{216}\right)}\right]^{-1}$$

($\tau_{3\to2}$ is the time characterizing the five-phonon scattering process and becomes comparable in this case with the time τ_{phr}).

The temperature dependences of the first-sound absorption coefficient calculated with formulas (1) - (6) for 1, 2.02, 3.91, and 6.08 Mc, are represented by the solid curves in Figs. a, b, c, and d. For comparison, the figures show the experimental data of Jefferson,



Temperature dependence of the first-sound absorption coefficient for the frequencies 1 (a), 2.02 (b), 3.91 (c), and 6.08 Mc (d) (o, ●, + -- data of Jeffers and Whitney; △, ▲ -- data of Chase for 2.00 and 6.00 Mc, respectively).

Whitney, and Chase [3]. The values of α_1 measured by these authors coincide with the theoretical values in a wide range of temperatures and frequencies.

We note that in the derivation of (1) - (6) we have assumed throughout that the phonon energy is $\epsilon = cp$. At very high frequencies, $\omega\tau_{phr}$, $\omega\tau_{phph} \gg [3\gamma(2\pi kT/c)^2(B_3/B_2)]^{-1}$ (B_2 and B_3 are Bernoulli numbers), the term containing γp^2 in the expression for ϵ ($\epsilon = cp(1 - \gamma p^2)$) can in general not be neglected. However, the good agreement between the theoretical values obtained with formula (3) and the experimental values of [3] offers evidence that γ is apparently much smaller than the value 3×10^{37} obtained from a rough extrapolation of the energy spectrum [4].

The investigation of the absorption of first sound at high frequencies presented in [5] was based on the assumption that the relative rate of establishment of equilibrium in the number of phonons and rotons is slow. No such situation is established in superfluid helium at the temperatures of greatest interest (below 1.2°K), and occurs only at higher temperatures.

In conclusion, the authors thank R. G. Mints, V. N. Sazonov, and D. Semiz for help during the numerical calculations.

[1] I. M. Khalatnikov, JETP 23, 21 (1952).

[2] I. M. Khalatnikov and D. M. Chernikova, JETP 49, 1957 (1965); 50, No. 2, (1966); translation in press.

[3] W. A. Jeffers and W. M. Whitney, Phys. Rev. 139, 1082 (1965); C. E. Chase, Ph. D. thesis, Cambridge University, 1954 (unpublished).

[4] L. D. Landau and I. M. Khalatnikov, JETP 19, 637 and 709 (1949).

[5] I. M. Khalatnikov, JETP 20, 243 (1950).

should read B. Ia. Zel'dovich and not Ia. B. Zel'dovich.

In the article "Dispersion of Sound in Superfluid Helium," by I. M. Khalatnikov and D. M. Chernikova, vol. 2, No. 12, p. 352 (Russ. p. 568), the formula in the 7th line from the bottom should read

$$u_{2\infty} = \{(\rho_{sr}/\rho_{nr})(\sigma_r^2/\partial\sigma_r/\partial T)_\rho)\}^{1/2},$$

formula (2) on p. 353 should read

$$\varphi = z_{phr} - 3\,\frac{u^2 \ln a + [2uz_{phr} + z_{phr}^2(1-\beta(1-z_{phr})) + 3u^2(1-z_{phr})][-2+(z_{phr}+z_{phph}-1)\ln a]}{2 + [1-z_{phph}+(1-\beta)(1-z_{phr})]\ln a + 3(1-z_{phr})[1-\beta(1-z_{phr})][-2+(z_{phr}+z_{phph}-1)\ln a]}$$

and a factor (ρ_{nph}/ρ) has been omitted after the braces in formula (6).