energy Physics, Dubna, in press.

[7] G. A. Smith, J. S. Lindsey, J. Button-Shafer, and J. J. Murray, Phys. Rev. Lett. 14, 25 (1965).

[8] V. P. Belov and V. M. Shekhter, YaF 2, 757 (1965), Soviet JNP, in press.

[9] V. Gupta and V. Singh, Phys. Rev. 135, B1442 (1964).

POSSIBLE REALIZATION OF NEGATIVE CONDUCTIVITY WITH NONEQUILIBRIUM CARRIERS IN SEMICONDUCTORS

V. F. Elesin and E. A. Manykin
Moscow Engineering Physics Institute
Submitted 15 November 1965
ZhETF Pis'ma 3, No. 1, 26-31, 1 January 1966

1. The energy distribution of nonequilibrium carriers (holes) usually differs little from the quasiequilibrium distribution, which is characterized by an effective temperature [1]. However, if the lifetime of the electron $\tau_e(\epsilon_{\vec{p}})$ in the conduction band is smaller than either the electron-electron interaction time or the time of relaxation on the acoustic phonons, then the electron distribution differs greatly from quasiequilibrium. The presence of such a nonequilibrium state can lead to several qualitatively new phenomena, in particular to negative conductivity, i.e., to the appearance of an electric current in a direction opposite to that of the external field $\vec{E}$. The effect of negative conductivity, which will be considered below, turns out to be connected with the threshold character of the interaction between the electrons and the optical phonons.

2. Let us consider a semiconductor at low temperatures ($kT \ll \hbar\omega_0$, ω_0 = frequency of the optical phonon) 1). Let the concentration of the equilibrium electrons be small compared with the concentration of the electrons produced in the conduction band under the influence of an external monochromatic source of intensity J with distribution $g(\Omega)$ (Fig. 1). The behavior of the electrons in the presence of an external field $\vec{E}$ is described by the usual kinetic equation:

$$-e\vec{E}\,\vec{\nabla}_{\vec{p}}\, f(\vec{p}) = \left(\frac{\partial f}{\partial t}\right)_{im} + \left(\frac{\partial f}{\partial t}\right)_{op} - \frac{f(\vec{p})}{\tau_e(\epsilon_{\vec{p}})} + Ig(\epsilon_{\vec{p}} - \omega), \qquad (1)$$

where $(\partial f/\partial t)_{im}$ = integral of elastic collisions with impurities, $(\partial f/\partial t)_{op}$ = integral of collisions with phonons. The term $f(\vec{p})/\tau_e(\epsilon_{\vec{p}})$ describes the recombination of the electrons, while $Ig(\epsilon_{\vec{p}} - \omega)$ describes the creation of electrons, I being connected with the intensity of the source J and with the absorption coefficient $k(\omega)$ of the semiconductor by the relation $I = Jk(\omega)\pi^2/(2\omega)^{1/2}m^{3/2}$.

In weak electric fields the solution of (1) is sought in the form

$$f(\vec{p}) = f_0(\epsilon_{\vec{p}}) + \frac{e\,\vec{E}\cdot\vec{p}}{m}\,\tau(\epsilon_{\vec{p}})\,\frac{\partial f_0(\epsilon_{\vec{p}})}{\partial t}, \qquad (2)$$

where

$$\tau^{-1}(\epsilon_{\vec{p}}) = \tau_e^{-1}(\epsilon_{\vec{p}}) + \tau_{im}^{-1}(\epsilon_{\vec{p}}) + \tau_{op}^{-1}(\epsilon_{\vec{p}}),$$

and $f_0(\epsilon_{\vec{p}})$ is the symmetrical part of the nonequilibrium stationary electron distribution function, satisfying the equation

$$f_0(\epsilon_{\vec{p}})[\tau_e^{-1}(\epsilon_{\vec{p}}) + \tau_{op}^{-1}(\epsilon_{\vec{p}})] = f_0(\epsilon_{\vec{p}} + \omega_0)\tau_{op}^{-1}(\epsilon_{\vec{p}})\left(\frac{\epsilon_{\vec{p}} + \omega_0}{\epsilon_{\vec{p}} - \omega_0}\right)^{1/2} + Ig(\epsilon_{\vec{p}} - \omega). \quad (3)$$

It is assumed here that $\epsilon_{\vec{p}} = p^2/2m$, and the frequency and the matrix element of the phonon-electron interaction do not depend on the quasimomentum $\vec{p}$. The probability of phonon emission takes the form

$$\tau_{op}^{-1}(\epsilon_{\vec{p}}) = A\sqrt{\epsilon_{\vec{p}} - \omega_0}, \quad A = \text{const}. \quad (4)$$

The solution of (3) can be obtained for arbitrary ω. For the interval of interest to us, $\omega_0 < \omega < 2\omega_0$, it takes the form

$$f_0(\epsilon_{\vec{p}}) = \frac{Ig(\epsilon_{\vec{p}} - \omega)}{\tau_{op}^{-1}(\epsilon_{\vec{p}}) + \tau_e^{-1}(\epsilon_{\vec{p}})} + \frac{Ig(\epsilon_{\vec{p}} + \omega_0 - \omega)\, A\, \tau_e(\epsilon_{\vec{p}})\sqrt{\epsilon_{\vec{p}} + \omega_0}}{\tau_{op}^{-1}(\epsilon_{\vec{p}} + \omega_0) + \tau_e^{-1}(\epsilon_{\vec{p}} + \omega_0)}. \quad (5)$$

Using (2), we write down the expression for the current in the form

$$\vec{j} = -\frac{e}{m}\int_0^\infty \frac{2d\vec{p}}{(2\pi^3)}\,\frac{e\vec{E}\vec{p}}{m}\,\tau(\epsilon_{\vec{p}})\,\frac{\partial f_0(\epsilon_{\vec{p}})}{\partial\epsilon_{\vec{p}}} = \vec{j}_+ + \vec{j}_-,$$

$$\vec{j}_+ = \vec{E}\,\frac{2e^2\sqrt{2m}}{\pi^2}\int_0^\infty d\epsilon\sqrt{\epsilon}\, f_0(\epsilon)[\tau(\epsilon) - \tfrac{2}{3}\epsilon\tau^2(\epsilon)\,\frac{d}{d\epsilon}\,(\tau_{im}^{-1} + \tau_e^{-1})], \quad (6)$$

$$\vec{j}_- = -\vec{E}\,\frac{4e^2\sqrt{2m}}{3\pi^2}\int_0^\infty d\epsilon\, \epsilon^{3/2} f_0(\epsilon)\,\tau^2(\epsilon)\,\frac{d}{d\epsilon}\,\tau_{op}^{-1}(\epsilon). \quad (7)$$

We have here integrated by parts and separated two current components, of which $\vec{j}_+$ is always positive (in the direction of $\vec{E}$) and $\vec{j}_-$, which, due to the dependence of τ_{op} on the energy, is negative.

Let us calculate the currents $\vec{j}_+$ and $\vec{j}_-$ under the assumption that the source width $\Gamma \ll \omega - \omega_0$. Taking outside the integral sign in (6) and (7) the functions that vary slowly compared with $f_0(\epsilon_{\vec{p}})$, we obtain, taking (5) into account,

$$j_+ = \frac{e\,E\,\tau^*(\omega)}{m}\, en_1 + \frac{e\,E\,\tau^*(\omega - \omega_0)}{m}\, en_2, \quad (8)$$

$$j_- = [\frac{2\omega}{m}]^{\frac{1}{2}}\, e\Delta n, \quad (9)$$

where we use the notation

$$\tau^*(\omega) = \tau(\omega) - \frac{2}{3}\,\omega\,\tau^2(\omega)\,\frac{d}{d\omega}\,(\tau_e^{-1} + \tau_{im}^{-1}),$$

$$n_1 = \frac{J\ k(\omega)\ \tau_e(\omega)}{1 + \tau_e(\omega)\ \tau^{-1}_{op}(\omega)}\ ; \qquad n_2 = \frac{J\ k(\omega)\ \tau_e(\omega - \omega_0)\ \tau_e(\omega)\ \tau^{-1}_{op}(\omega)}{1 + \tau_e(\omega)\ \tau^{-1}_{op}(\omega)}\ ,$$

$$\Delta n = \frac{\sqrt{2}}{3}\ n_1 \tau^2(\omega)\ e\ E \sqrt{\frac{\omega}{m}}\ \frac{d}{d\omega}\ \tau^{-1}_{op}(\omega),$$

the physical meaning of which is as follows: $\tau^*(\omega)$ characterizes the time of acceleration of the electron in the field $\vec{E}$, n_1 and n_2 are the densities of electrons with energies near ω and $\omega - \omega_0$, and Δn is the density of the electrons that contribute to the negative conductivity. Inasmuch as $\tau^*(\omega) \approx \tau(\omega)$, we have for all the current ratios

$$j_+/j_- = 3(\omega - \omega_0)^{1/2}[\tau(\omega)\ \omega\ A]^{-1} + 3(\omega - \omega_0)\ \omega^{-1}\ \tau_e(\omega - \omega_0)\ \tau(\omega - \omega_0)\ \tau^{-2}(\omega). \tag{10}$$

We see therefore that when ω approaches ω_0, this ratio decreases and becomes smaller than unity under the condition

$$\left[\frac{\omega - \omega_0}{\omega}\right]^{1/2} < \frac{2}{3}\ \tau(\omega)\ A\ \sqrt{\omega}\ \frac{\sqrt{1 + \xi} - 1}{\xi}$$

where

$$\xi = \frac{4}{3}\ \tau(\omega - \omega_0)\ \tau_e(\omega - \omega_0)\ A^2\ \omega.$$

For the more realistic case when $\xi \gg 1$, we obtain

$$\frac{\omega - \omega_0}{\omega} < \frac{\tau^2(\omega)}{3\tau_e(\omega - \omega_0)\ \tau(\omega - \omega_0)}\ .$$

For example, for InSb in this case

$$\tau(\omega) \approx \tau_{im}(\omega) = B\ \omega^{3/2} \sim 10^{-12}\ \ \text{sec},$$

and $\tau_e \sim 10^{-10}$ sec [3], so that the negative conductivity should take place when $\omega - \omega_0 < (0.01 - 0.1)\omega$.

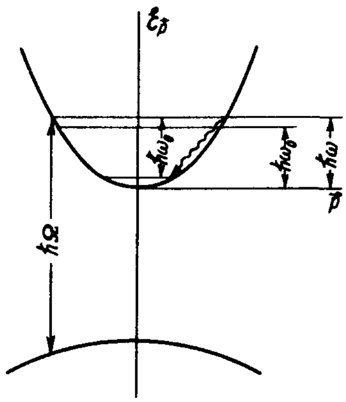

Fig. 1.

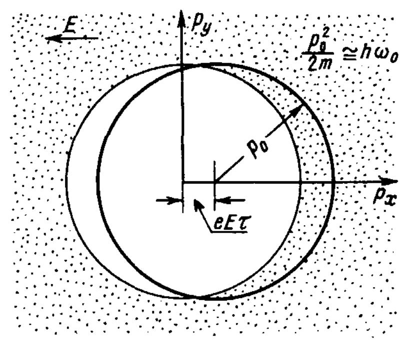

Fig. 2. Shaded area - region of interaction of electrons with optical phonons.

3. The effect of negative conductivity has a simple physical interpretation. In phase space the electrons are produced in a narrow spherical layer near the sphere $p^2/2m = \omega \approx \omega_0$

(Figs. 1 and 2). Application of an external electric field shifts the sphere by $e\vec{E}\tau(\omega)$, so that a fraction of the electrons (on the right hemisphere) falls into the region of more effective interaction with the phonons, compared with the electrons on the left hemisphere, and the electrons lose practically all their momentum and energy as a result of phonon emission. This gives rise to a directed flux of electrons along the field, with a larger momentum $p \sim \sqrt{m\omega}$, which in turn gives rise to the negative current $\vec{j}_-$ (see (9)).

4. Our analysis is valid in electric fields satisfying the inequality $eE(\omega)\sqrt{\omega/m} \ll \omega - \omega_0$. For sufficiently large fields this condition does not hold, so that the solution of the problem calls for a separate analysis in this case. However, recognizing that the electric field causes a broadening of the electron distribution by $eE\tau(\omega)\sqrt{\omega/m}$, the results obtained above remain qualitatively valid if we replace in them $\omega - \omega_0$ by $eE\tau(\omega)\sqrt{\omega/m}$.

5. We note that the assumptions that the electron dispersion is quadratic, and that the matrix element of the electron-phonon interaction is independent of the quasimomentum, are not important and have been made only to simplify the derivation. The effect of negative conductivity is due only to the strongly unbalanced distribution of the electron energies and to the threshold character of the interaction between the electrons and the phonons, so that analogous phenomena can take place also in elastic collisions between electrons and atoms in gases.

The authors are deeply grateful to A. M. Afanas'ev for continuous help, and to N. G. Basov, Yu. A. Bykovskii, V. M. Galitskii, and Yu. M. Kagan for an evaluation of the work.

[1] S. M. Ryvkin, Fotoelektricheskie yavleniya v poluprovodnikakh (Photoelectric Phenomena in Semiconductors), Fizmatgiz, 1963, p. 22.
[2] Yu. A. Bychkov and A. M. Dykhne, JETP 48, 1168 (1965), Soviet Phys. JETP 21, 779 (1965).
[3] M. A. Habegger and H. Y. Fan, Phys. Rev. Lett. 12, 99 (1964).

1) We put $\hbar = 1$ throughout, and the word "phonon" in the text stands for optical phonon.

EXPERIMENTAL OBSERVATION OF FOURTH SOUND IN He^3-He^4 SOLUTIONS

B. N. Esel'son, N. E. Dyumin, E. Ya. Rudavskii, and I. A. Serbin
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
Submitted 15 November 1965
ZhETF Pis'ma 3, No. 1, 32-35, 1 January 1966

It has been shown theoretically [1-2] and experimentally [3-4] that liquid He^4 can support fourth sound - a special type of wave propagating only through the superfluid component, while the normal component remains immobile. Fourth sound is realized in narrow channels with characteristic transverse dimensions much smaller than the depth of penetration of the viscous wave, $d \ll (2\eta_n/\omega\rho_n)^{1/2}$, where ρ_n and η_n are the density and viscosity of the normal component, respectively, and ω is the oscillation frequency.

The question of existence of waves of this type in He^3-He^4 solutions was recently considered theoretically [5]. It follows from that paper that fourth sound should be observable