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JUMP IN SPECIFIC HEAT ON GOING FROM THE SUPERCONDUCTING TO THE NORMAL STATE

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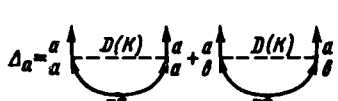
The ratio $C_s(T_k)/C_n(T_k) = 2.4$ (C_s and C_n are the electronic specific heats in the superconducting and normal states, respectively), was obtained in the usual theory of superconductivity [1] by considering an anisotropic model. Account of anisotropy [2] leads, as is well known, to the inequality $\alpha = C_s(T_k)/C_n(T_k) < 2.4$, which is not satisfied in the case of many superconductors (a survey of the experimental data can be found, for example, in [3]). An analysis within the framework of the Frohlich model [4], in which the electron-phonon interaction is described more accurately, makes this inequality even stronger.

This paper is devoted to an investigation of the magnitude of the jump in specific heat from the point of view of the two-band model. The need for such investigation is dictated by the fact that superconductors for which the inequality $\alpha < 2.4$ is not satisfied are characterized by the presence of overlapping bands (see below). The two-band model was first developed in [5,6] on the basis of the u,v-transformation method.

In the presence of two overlapping bands (a and b) it is necessary to consider two self-energy parts Δ_a and Δ_b which determine the values of the corresponding energy gaps and which satisfy, in diagram form, the equations:



(1)

(2)

where

$$F^{+}_{aa,bb} = \Delta_{a,b}/(\omega_n^2 + \xi^2_{a,b} + \Delta^2_{a,b}).$$

We disregard terms containing $\Delta_{a,b}$ and $F^{+}_{a,b}$ and describing the pairing of electrons be-

longing to different bands, in view of the slight probability of an overlap whereby electrons with opposite quasimomenta appear near the Fermi surface in different bands.

The entropy corresponding to two branches of excitations can be written for $T \to T_k$ in the form

$$S = S_a + S_b = (\pi^2/3)T(\nu_a + \nu_b) - (1/2)(\nu_a \frac{\Delta_a^2}{T} + \nu_b \frac{\Delta_b^2}{T}) \quad (3)$$

(ν_a and ν_b are the state densities in bands a and b, respectively; for an analogous calculation for the single-band model see [7]).

The solution of (1) and (2) allows us to determine the critical temperature T_k. From the same equations, expanding the functions F_{aa}^+ and F_{bb}^+ in powers of Δ_a and Δ_b, we can determine the functions Δ_a/T and Δ_b/T as $T \to T_k$, and obtain, after substituting the appropriate expressions in (3), the following expression for the jump of specific heat:

$$\frac{C_s(T_k)}{C_n(T_k)} = 1 + 1.4\ F, \quad (4)$$

$$F = \frac{(g_{aa}\nu_a^2/\lambda_0^2) + g_{bb}\nu_b^2 + 2g_{ab}\nu_a\nu_b/\lambda_0}{(g_{aa}\nu_a^2/\lambda_0^2) + g_{bb}\nu_b^2\lambda_0^2 + g_{ab}\nu_a\nu_b(\lambda_0 + \lambda_0^{-1})} \cdot \frac{\nu_a + \nu_b\lambda_0^2}{\nu_a + \nu_b}.$$

Here g_{aa} and g_{bb} are the constants of the electron-phonon interaction that describes the electron transitions in the a and b bands, respectively, g_{ab} corresponds to interband transitions, and

$$\lambda_0 = (\Delta_b/\Delta_a)_{T=T_k}\ ;\quad \lambda_0 = \frac{g_{bb}\nu_b - g_{aa}\nu_a + \sqrt{(g_{bb}\nu_b - g_{aa}\nu_a)^2 + 4g_{ab}^2\nu_a\nu_b}}{2g_{ab}\nu_b}.$$

It is seen from (4) that the equality $\alpha = 2.4$ is, generally speaking, not satisfied in the two-band model (in real cases $\lambda_0 \neq 1$). It is, of course, impossible to calculate the numerical value of the jump, since this would call for knowledge of the constants g_{aa}, g_{bb}, and g_{ab}. We see, however, that values $C_s/C_n > 2.4$ are perfectly admissible. It is necessary for this purpose to satisfy the perfectly natural conditions $\nu_b > \nu_a$ and $\lambda_0 > 1$, i.e., that the narrower band b correspond to a larger value of the energy gap.

The band overlap for superconducting elements is therefore not the exception but the rule; in particular, all the elements for which the experimental value is $\alpha > 2.4$ have a non-single-band structure. In fact, this includes Nb ($\alpha = 3.07$), Ta ($\alpha = 2.58$), and V ($\alpha = 2.57$), which are transition metals. While Sn ($\alpha = 2.6$), Al ($\alpha = 2.6$), and In ($\alpha = 2.65$) are not transition elements, they exhibit a noticeable band overlap (cf., e.g., [8]).

It is shown in [10] that the band overlap effect leads to a dependence of T_k on the concentration of the nonmagnetic impurities (at low concentrations $\delta T_k \sim l^{-1}$, where l is the mean free path). Such a dependence was observed experimentally for Sn, Al, Ta, and In [9], and likewise confirms the statements made above concerning the character of the band structure in these elements. This relation is not observed for Tl ($\alpha = 2.15$), and it is therefore natural for the condition $\alpha < 2.4$, which holds for the single-band model, to be satisfied for

it. The overlap effect is quite small also for Zn ($\alpha = 2.25$), Cd ($\alpha = 2.2$), and Ga ($\alpha = 2.4$), for which $\alpha \leq 2.4$.

In the presence of two gaps, a deviation is observed in the low-temperature region of the dependence of the specific heat from the ordinary exponential dependence. As $T \to 0$ the main contribution to the specific heat is made, naturally, by the smaller gap, and this leads to a certain slowing down in the drop of the function $C_s(T)$. This is observed in Nb, V, and Ta [11]. It is possible that in the case of Pb, for which the experimental data at $T \to 0$ can be attributed to the presence of two gaps ($\Delta_1 = 2.05T_k$ and $\Delta_2 = 0.55T_k$), the existence of two gaps is connected not with anisotropy, as assumed in [12], but with band overlap (according to [8] Pb has three overlapping bands). The main role in the absorption of electromagnetic waves is played by the gap Δ_1, but it is possible that the fine structure of the absorption spectrum is also connected with the presence of the second gap [13].

We see therefore that the temperature variation of the specific heat changes appreciably in the presence of overlapping energy bands. The magnitude of the jump in specific heat on going from the superconducting state to the normal phase is thus likewise essentially different. The non-single-band model makes possible larger values of $C_s/C_n\big|_{T = T_k}$ than in the isotropic single-band model.

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ERRATUM

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Article by B. T. Geilikman and V. Z. Kresin, "Jump in Specific Heat on Going from the Superconducting to the Normal State." In line 3 of the first paragraph "by considering an anisotropical model" should read "by considering an isotropical model."