NONLINEAR POLARIZATION OF RADIATION PASSING THROUGH A PLASMA

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We shall show that nonlinear interaction effects can noticeably alter the polarization of radiation passing through a plasma, if the radiation has sufficient intensity or if its path in the plasma is sufficiently long. Our results can be used in investigations of the polarization properties of cosmic radiation [1], propagation of radio waves, etc.

Assuming that the nonlinear effects are weak, we can expand the current produced by the wave in the plasma in powers of the wave amplitude. Since a current that is quadratic in the field amplitude describes, as is well known [2], only interactions between transverse and longitudinal waves 1), we shall concern ourselves with a current $\vec{j}^{(3)}$ containing the third power of the wave amplitudes. The wave propagation equation takes the form

$$(K^2 - \omega^2\epsilon^{t2})\vec{E}_{K\omega} = \frac{e^2\omega_{oe}^2\omega}{2m_e^2}\int d\lambda \frac{(\vec{K}_2 + \vec{K}_3)^2}{(\omega_2 + \omega_3)^2} \vec{E}_{K_1\omega_1}(\vec{E}_{K_2\omega_2}\cdot\vec{E}_{K_3\omega_3}). \quad (1)$$

Here $\omega_{oe}^2 = 4\pi e^2 N/m_e$, $\vec{E}_{K\omega}$ are the Fourier components of the wave field, ϵ the linear dielectric constant of the plasma, and $d\lambda = dK_1 dK_2 dK_3 \delta(K - K_1 - K_2 - K_3)$, where $K = \{\vec{K}\omega\}$. Since the nonlinearity is weak, a solution of (1) can be sought by the method of Bogolyubov and Van der Pol [4] (cf. also [6,7], assuming that

$$\vec{E}(\vec{r},t) = \int \vec{E}_K(\vec{r},t)\exp[i(\vec{K}\cdot\vec{r} - \omega(\vec{K})t)]\, d\vec{K}; \quad \vec{E}_K = \vec{E}^x_{-K}; \quad \omega(\vec{K}) = -\omega(-\vec{K}),$$

where $\omega(\vec{K})$ is the solution of the linear dispersion equation and $\vec{E}_K(\vec{r}, t)$ is an amplitude that varies slowly in time and in space (compared with $1/\omega$ and $1/|\vec{K}|$). Retaining only the first derivatives of the amplitude in the left side of (1) and neglecting the variation of $\vec{E}_K(\vec{r}, t)$ with $\vec{r}$ or t in the right side, we obtain

$$\left(\frac{\partial}{\partial t} + \vec{v}_{gr}\frac{\partial}{\partial t}\right)\vec{E}_K(\vec{r}, t) = \frac{1}{i}\frac{e^2\omega_{oe}^2}{\left|\frac{\partial}{\partial\omega}\omega^2\epsilon^t\right|}\frac{1}{2m_e^2}\int\frac{d\vec{K}_1 d\vec{K}_2 d\vec{K}_3}{\omega(\vec{K}_1)\omega(\vec{K}_2)\omega(\vec{K}_3)}$$

$$\times \exp\{it[\omega(\vec{K}) - \omega(\vec{K}_1) - \omega(\vec{K}_2) - \omega(\vec{K}_3)]\}\,\delta(\vec{K} - \vec{K}_1 - \vec{K}_2 - \vec{K}_3)[\omega(K_1) + \omega(\vec{K}_2) + \omega(\vec{K}_3)] \quad (2)$$

$$\times \frac{(\vec{K}_2 + \vec{K}_3)^2\vec{E}_{K_1}(\vec{r}, t)}{[\omega(\vec{K}_2) + \omega(\vec{K}_3)]^2}(\vec{E}_{K_2}(\vec{r}, t)\,\vec{E}_{K_3}(\vec{r}, t).$$

Here $\vec{v}_{gr} = (\vec{K}/K)(d\omega/dK)$ is the group velocity of the waves in the linear approximation (we neglect the linear damping of the waves).

2. Let us consider the interaction between different polarization components of a single monochromatic wave with $\omega \gg \omega_{oe}$

$$\vec{E}_K = \vec{E}\delta(\vec{K} - \vec{K}_0) + \vec{E}^x\delta(\vec{K} + \vec{K}_0),$$

where $\vec{K}_0$ is the wave vector of the monochromatic wave. Retaining only terms satisfying the phase-synchronism conditions $\omega(\vec{K}) - \omega(\vec{K}_1) - \omega(\vec{K}_2) - \omega(\vec{K}_3) = 0$, we obtain

$$\frac{d\vec{E}}{dt} = i\beta\vec{E}^x(\vec{E}^2); \quad \beta = \frac{e^2\omega_{oe}^2}{4m_e^2\omega^3}. \tag{3}$$

Writing the polarization components E_1 and E_2 (the projections of $\vec{E}$ on two directions perpendicular to $\vec{K}_0$) in complex form, $E_1 = A_1 \exp(i\varphi_1)$ and $E_2 = A_2 \exp(i\varphi_2)$, we obtain equations for A_1, A_2, and $\varphi = \varphi_1 - \varphi_2$:

$$\frac{\partial A_1}{\partial t} = -\beta A_1 A_2^2 \sin 2\varphi; \quad \frac{\partial A_2}{\partial t} = \beta A_2 A_1^2 \sin 2\varphi; \quad \frac{\partial\varphi}{\partial t} = \beta(A_1^2 - A_2^2)(1 - \cos 2\varphi). \tag{4}$$

The system (4) has two integrals - energy conservation $A_1^2 + A_2^2 = A_1^2(0) + A_2^2(0)$, and angular-momentum conservation [7] $A_1A_2\sin\varphi = A_1(0)A_2(0)\sin^2\varphi_0$. These make it possible to obtain the solution. The result is simplest in form if the directions 1 and 2 are chosen to be of the principal axes of the polarization ellipse at $t = 0$, i.e., $\varphi_0 = \pm\pi/2$ [2)]

$$A_1^2 = A_1^2(0)\cos^2\tau + A_2^2(0)\sin^2\tau; \quad A_2^2 = A_1^2(0)\sin^2\tau + A_2^2(0)\cos^2\tau, \tag{5}$$

$$\tau = 2\beta t A_1(0)A_2(0). \tag{6}$$

In this case most of the energy will have already been "pumped over" within a time $\tau = \pi/2$ into the component A_2. The lengths $b_{1,2}$ of the two semi-axes of the polarization ellipse at each instant of time can be expressed in terms of the invariants,

$$b_{1,2}(t) = \frac{1}{2}([A_1^2 + A_2^2 + 2A_1A_2 \sin\varphi]^{1/2} \pm [A_1^2 + A_2^2 - 2A_1A_2 \sin\varphi]^{1/2}) = b_{1,2}(0),$$

i.e., the ellipse does not change in shape, but merely rotates.

It is easy to ascertain that the angle θ between the major semi-axis and its direction at $t = 0$ increases linearly in time, $\tan 2\theta = \pm\tan 2\theta$. The minus sign corresponds to $\varphi_0 = \pi/2$, and plus to $\varphi_0 = -\pi/2$, i.e., the direction of the nonlinear rotation of the ellipse is opposite to the electric vector rotation in the ellipse. The characteristic rotation time depends on the product of the initial energy densities W_1 and W_2 along the principal axes, and its estimate is

$$t \simeq \frac{1}{2\beta A_1(0)A_2(0)} = \frac{2}{\omega_{oe}}\left(\frac{\omega}{\omega_{oe}}\right)^3 \frac{nm_e c^2}{(W_1W_2)^{1/2}}. \tag{7}$$

It can also be shown that when two waves having different frequencies interact the sum of the energies of the two components of each of the polarizations is conserved, i.e., the interaction (3) changes only the wave polarization.

3. If the interacting waves are random, then we can easily obtain from (3) equations for the mean value of the polarization components E_{1K} and E_{2K} [3])

$$\langle E_{1K}\, E_{1K}\rangle = |E_{1K}|^2 \delta(\vec{K} + \vec{K}') = A_{1K}\delta(\vec{K} + \vec{K}'),$$

$$\langle E_{2K}\, E_{2K}\rangle = |E_{2K}|^2 \delta(\vec{K} + \vec{K}') = A_{2K}\delta(\vec{K} + \vec{K}'),$$

$$\langle E_{1K}\, E_{2K}\rangle = \langle E_{1K}\, E_{2K}^{x}\rangle \delta(\vec{K} + \vec{K}') = B_K\delta(\vec{K} + \vec{K}').$$

The four coefficients A_{1K}, A_{2K} and $B_K = \mathrm{Re}B_K + i\ \mathrm{Im}B_K$ can be easily related to the Stokes parameters [7]. We have

$$\frac{dA_{1K}}{dt} = -\frac{\partial A_{2K}}{\partial t} = -\frac{2e^2\omega_{oe}^2}{m_e^2\left|\frac{\partial}{\partial\omega}\omega^2\epsilon\right|_{\omega=\omega(\vec{K})}} \mathrm{Im}\int \frac{d\vec{K}_1}{\omega^2(\vec{K}_1)} \frac{(\vec{K} - \vec{K}_1)^2}{[\omega(\vec{K}) - \omega(\vec{K}_1)]^2} [B_K^x B_{K_1} pq + B_K B_{K_1}^x rs + A_{1K_1} B_K^x ps + A_{2K_1} B_K^x qr] \tag{8}$$

$$\frac{\partial B_K}{\partial t} = \frac{ie^2\omega_{oe}^2}{m_e^2\left|\frac{\partial}{\partial\omega}\omega^2\epsilon^t\right|_{\omega=\omega(\vec{K})}} \int \frac{d\vec{K}_1(\vec{K} - \vec{K}_1)^2}{\omega^2(\vec{K}_1)[\omega(\vec{K}) - \omega(\vec{K}_1)]^2} \{B_K[A_{1K_1}(p^2 - s^2) + A_{2K_1}(r^2 - q^2) + (B_K + B_{K_1}^x)(rp - sq) + (A_{2K} - A_{1K})][B_{K_1}pq + B_{K_1}^x rs + A_{rK_1}qr + A_{1K_1}ps]\}, \tag{9}$$

where

$$p = (\vec{e}_{1K}\cdot\vec{e}_{1K_1}), \quad q = (\vec{e}_{2K}\cdot\vec{e}_{2K_1}), \quad r = (\vec{e}_{2K_1}\cdot\vec{e}_{1K}), \quad s = (\vec{e}_{2K}\cdot\vec{e}_{1K_1}).$$

The first equation in (8) shows that the interaction of the wave $\vec{K}$ with any other wave can lead only to a change in the ratio of the intensities of the different polarization components, without changing the total energy of the wave $\vec{K}$. This characteristic property of the nonlinear interaction in question sharply distinguishes it from other known nonlinear interactions (decay processes and induced scattering) that lead to a change in the spectral composition of the radiation. Another consequence of the first equation in (8) is the conservation of entropy (which follows from the conservation of the number of quanta), and consequently reversibility of the nonlinear interactions for random waves. The latter can be illustrated by using as examples the solutions of (8) and (9) for almost-monochromatic waves [4])

$$A_1(t) = A_1(0)\cos^2\frac{\Omega t}{2} + A_2(0)\sin^2\frac{\Omega t}{2};$$

$$A_2(t) = A_1(0)\sin^2\frac{\Omega t}{2} + A_2(0)\cos^2\frac{\Omega t}{2},$$

$$\mathrm{Re}\,B = \mathrm{Re}\,B(0) - \frac{e^2\omega_{oe}^2\,\mathrm{Im}\,B(0)}{m_e^2\omega^3}\sin\Omega t, \quad \mathrm{Im}\,B = \mathrm{Im}\,B(0),$$

$$\Omega = 2\frac{e^2\omega_0^2}{m_e^2\omega^3}\,\mathrm{Im}\,B(0), \qquad A_{1,2} = \int A_{1,2K}d_K, \quad B = \int B_K d_K.$$

This solution is similar to that obtained for a fixed phase, and leads to the same estimates as in (7).

4. In conclusion we present a rough estimate, which illustrates the role of the interaction under consideration for the most unfavorable case, when the ellipticity is quite small (e.g. in (7)). Assuming that the dimension L of the object is equal to the path ct on which the ellipse rotates through $\pi/2$, we can estimate the minimum degree of ellipticity W_1/W_2, starting with which the effect becomes significant. For the Crab nebula, for example, with $W_1 = W_2 = 5$ eV/cm^3 at $\lambda = 2\pi c/\omega \sim 100$ m, $L = 10^{19}$ cm, and $n = 10$ cm^{-3}, we obtain the rather small value $W_1/W_2 \sim 10^{-3}$. An account of this effect becomes even more important for other radio sources with larger emission density, and also in the case of radio wave propagation in the ionosphere, etc.

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1) See [3,4] concerning astrophysical consequences of the decay processes described by this current.

2) We assume here for simplicity that all quantities depend only on t. It is easy to go over to the case when all quantities depend on x, by substituting x/c for t.

3) $\vec{E}_K = \vec{e}_{1K}E_{1K} + \vec{e}_{2K}E_{2K}$, where $\vec{e}_{1K}$ and $\vec{e}_{2K}$ are two unit polarization vectors perpendicular to $\vec{K}$.

4) For random waves, the characteristic time τ of nonlinear interaction should satisfy the condition $\tau \gg 1/\Delta\omega$, where $\Delta\omega$ is the width of the wave spectrum.

INVESTIGATION OF ELECTRON COLLISIONS WITH EXCITED NEON ATOMS

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To solve various physical problems connected with high-temperature nonequilibrium plasma (low-pressure gas discharge, lasers, ionosphere, astrophysical problems, etc.) it is necessary to know the effective cross sections for excitation and de-excitation of the atoms as they collide with electrons. The available experimental data pertain exclusively to transitions from