ON A POSSIBLE METHOD OF VERIFYING CPT INVARIANCE IN pp SCATTERING

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The invariance of an interaction with respect to the CPT transformation is the consequence of the basic postulates of modern field theory (the Luders-Pauli theorem [1,2]). An experimental check of CPT invariance is therefore also a verification of these postulates. Gotow and Okubo [3] were the first to call attention to the fact that a unique possibility of directly verifying CPT invariance of strong interaction is provided by a study of the polarization effects in elastic scattering of antiprotons by protons

$$\bar{p} + p \rightarrow \bar{p} + p. \qquad (1)$$

The authors of [3], however, did not obtain relations between the simplest observed quantities, an experimental verification of which would constitute a check on CPT invariance. We shall show here that CPT invariance implies the following relations:

$$\vec{P}^{(1)}\cdot\vec{n} = \vec{A}^{(2)}\cdot\vec{n},$$
$$\vec{P}^{(2)}\cdot\vec{n} = \vec{A}^{(1)}\cdot\vec{n}, \qquad (2)$$

where $\vec{n} = (\vec{p} \times \vec{p}\,')/(|\vec{p} \times \vec{p}\,'|)$ is the normal to the scattering plane ($\vec{p}$ and $\vec{p}\,'$ are the momenta of the initial and final antiprotons in the c.m.s.), $\vec{P}^{(1)}$ ($\vec{P}^{(2)}$) is the antiproton (proton) polarization produced upon scattering of unpolarized particles, and $\vec{A}^{(1)}$ ($\vec{A}^{(2)}$) is the asymmetry occurring when a polarized antiproton beam is scattered by an unpolarized target (unpolarized beam is scattered by polarized proton target). We denote the c.m.s. scattering matrix by $M(\vec{p}\,', \vec{p})$. Then, as is well known,

$$P_i^{(1)} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,\sigma_{1i}\,MM^+, \qquad P_i^{(2)} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,\sigma_{2i}\,MM^+,$$
$$A_i^{(1)} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,M\,\sigma_{1i}\,M^+, \qquad A_i^{(2)} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,M\,\sigma_{2i}M^+, \qquad (3)$$

where σ_0 is the differential cross section for scattering in the c.m.s. The spin matrices of the antiproton and proton are denoted by σ_{1i} and σ_{2i}, respectively. If the interactions causing the scattering process (1) are invariant against CPT transformation, then the scattering matrix satisfies the requirement

$$M^T(\vec{p}\,', \vec{p}) = U^{-1}\,P(1,2)\,M(-\vec{p}, -\vec{p}\,')\,P(1,2)\,U. \qquad (4)$$

Here $P(1, 2) = 1/2(1 + \vec{\sigma}_1\cdot\vec{\sigma}_2)$ is the spin-variable permutation operator, U is a unitary matrix satisfying the conditions

$$U^{-1}\,\sigma_{1i}\,U = -\sigma_{1i}^T,$$
$$U^{-1}\,\sigma_{2i}\,U = -\sigma_{2i}^T, \qquad (5)$$

and the symbol T stands for the transpose.

By rotation through an angle π around the vector $\vec{m} = (\vec{p}' - \vec{p})/|\vec{p}' - \vec{p}|$ and using the invariance to rotation, we obtain

$$M(-\vec{p}, -\vec{p}) = (\vec{\sigma}_1\cdot\vec{m})(\vec{\sigma}_2\cdot\vec{m})M(\vec{p}', \vec{p})(\vec{\sigma}_1\cdot\vec{m})(\vec{\sigma}_2\cdot\vec{m}). \tag{6}$$

With the aid of (3) - (6) we can easily obtain relations (2), which are the consequences only of the invariance against CPT transformation and invariance against rotation. In the general case, when the invariance of the S-matrix against CPT transformation is not assumed, the scattering matrix can be represented in the form

$$M(\vec{p}', \vec{p}) = M_1(\vec{p}', \vec{p}) + M_2(\vec{p}', \vec{p}), \tag{7}$$

where

$$\begin{aligned} M_1^T(\vec{p}', \vec{p}) &= U^{-1} P(1,2)\, M_1(-\vec{p}, -\vec{p}')\, P(1,2)\, U, \\ M_2^T(\vec{p}', \vec{p}) &= -U^{-1} P(1,2)\, M_2(-\vec{p}, -\vec{p}')\, P(1,2)\, U. \end{aligned} \tag{8}$$

We then obtain in lieu of (2)

$$\begin{aligned} \sigma_0(\vec{P}^{(1)}\cdot\vec{n} - \vec{A}^{(2)}\cdot\vec{n}) &= \mathrm{Re\,Sp}\, \vec{\sigma}_1\cdot\vec{n}\, M_1 M_2^+ \\ \sigma_0(\vec{P}^{(2)}\cdot\vec{n} - \vec{A}^{(1)}\cdot\vec{n}) &= \mathrm{Re\,Sp}\, \vec{\sigma}_2\cdot\vec{n}\, M_1 M_2^+. \end{aligned} \tag{9}$$

Violation of relations (2) would signify that the S-matrix is not invariant against CPT transformation. If it were to turn out that within the limits of experimental error the polarization $\vec{P}^{(1)}\cdot\vec{n}$ and the asymmetry $\vec{A}^{(2)}\cdot\vec{n}$ (or $\vec{P}^{(2)}\cdot\vec{n}$ and $\vec{A}^{(1)}\cdot\vec{n}$) coincide, then it is possible to determine with the aid of (9) the upper limit of the amplitude which is invariant to CPT transformation. Experiments aimed at verifying C(T) invariance of strong interactions show [4] that the upper limit of the ratio of the amplitude invariant to charge conjugation C, to the C-invariant amplitude is of the order of 1% (for T of the order of 2 - 3%). The recently observed parity nonconservation effects in nuclear reactions are compatible with the assumption that parity nonconservation is connected only with weak interactions [4]. Thus, a check on relations (2) calls for the setting up of difficult experiments in which the polarization and asymmetry would have to be measured with high accuracy. We note in conclusion that with the aid of (4) and (6) it is easy to obtain relations between other observed quantities. Let us present some of them:

$$\begin{aligned} D_{\ell m}^{(1)} &= -D_{m\ell}^{(2)}, \quad D_{m\ell}^{(1)} = -D_{\ell m}^{(2)}, \\ K_{\ell m}^{(1)} &= -K_{m\ell}^{(2)}, \quad K_{\ell m}^{(2)} = -K_{m\ell}^{(2)}, \\ C_{\ell m} &= -P_{m\ell}, \quad C_{m\ell} = -P_{\ell m}. \end{aligned} \tag{10}$$

For a determination of the quantities contained therein cf., e.g., [5].

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MOTION OF VORTEX FILAMENTS IN SUPERCONDUCTING ALLOYS

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It is difficult to apply the ordinary hydrodynamic approach [1] to the motion of vortex filaments in superconductors of type II, owing to the presence of the lattice. In the state of local equilibrium, the condensate is characterized by a superconducting velocity $\vec{v}_s$ and an electron density N. However, in view of the Coulomb interaction, the relaxation rate of N is so large that the only "hydrodynamic" quantity is $\vec{v}_s$. It is easy to obtain for $\vec{v}_s$ the equation

$$\dot{\vec{v}}_s = e/m\,(\vec{E} - \nabla\varphi),$$

but this is a trivial consequence of Maxwell's equations, since by definition

$$\mathrm{curl}\,\vec{v}_s + (e/m)\vec{H} = 0$$

($\vec{v}_s = 1/2\,m(\nabla x - 2e\vec{A})$, x is the phase of the ordering parameter, $\Delta = |\Delta|\exp(ix)$, $\vec{H} = \mathrm{curl}\,\vec{A}$, and $\hbar = c = 1$).

In view of the field character of the vortex, being a line of singularities of $v_s \sim 1/2mr$ [2], for the solution of the problem it is in fact sufficient to find directly the response of the superconducting system to an external field, i.e., the current $\vec{j} = \vec{j}\{\vec{A}\}$ as a functional of $\vec{A}$. In the case of "dirty" alloys ($\tau T_c \ll 1$), for physically reasonable frequencies $\omega\tau \ll 1$, this can be done by directly expanding $\vec{j}$ in terms of the small parameter $\omega\tau$:

$$\vec{j} = \vec{j}_s\{\vec{v}_s\} + \vec{j}_n, \qquad \vec{j}_n = \hat{\sigma}\{\vec{v}_s\}\vec{E},$$

where $\vec{j}_s\{\vec{v}_s\}$ is the equilibrium superconducting current in the specified magnetic field $\vec{A}$. The phase x in $\vec{v}_s$ is determined by the additional equation div $\vec{j}_s = 0$, which follows from the imaginary part of the equilibrium equation for the ordering parameter. The kernel of the conductivity operator $\hat{\sigma}\{\vec{v}_s\}$ can be shown to have the form

$$(-i)d/d\omega\,\langle j_i, j_k\rangle\big|_{\omega\to\Omega},$$

where the correlator $\langle j_i, j_k\rangle$ can also be calculated from the equilibrium state in the specified external field [1)]. The relations presented constitute, together with Maxwell's equations and the continuity equation, a complete system that yields the vortex equation of motion.