MASS FORMULAS IN WEAKLY BROKEN SU(3) SYMMETRY

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We propose in this letter a new derivation of the relation between the masses of arbitrary-spin particles belonging to different SU(3) multiplets. We start from the following assumptions:

I. The three-dimensional integrals of the time-dependent components of the vector current ($Q_\alpha^V(t) = \int d^3x\, I_0^\alpha(x,t)$) are generators of the SU(3) algebra [1], i.e.,

$$[Q_\alpha^V(t),\ Q_\beta^V(t)] = if_{\alpha\beta\gamma}\ Q_\gamma^V(t) \tag{1}$$

and transform single-particle states into single-particle states that are nearest in energy. An essential factor here is the use of the covariant formalism of [2]. This approximation corresponds to the premise that definite multiplets of SU(3) exist in spite of the inequality of the corresponding particle masses.

II. There exists a 4-vector $L_\mu^\alpha(x,t)$, transforming in accordance with the octet representation of SU(3), the divergence of which satisfies the commutation relation

$$[\frac{\partial}{\partial x_\mu}\ L_\mu^{K^+}(x,t),\ Q_{K^+}^V(t)] = 0. \tag{2}$$

Within the framework of exact SU(3), Eq. (2) follows from the fact that L_μ^α transforms in accordance with the octet representation of SU(3), i.e.,

$$[L_\mu^\alpha(x,t),\ Q_\beta^V(t) = if_{\alpha\beta\gamma}\ L_\mu^\gamma(x,t). \tag{3}$$

An example of an operator L_μ^α satisfying assumption II in the approximate symmetry is the vector current in a model in which the breaking of SU(3) in the Hamiltonian transforms like T_3^3 [2], or the axial current in the quark model [3] or in the hypothesis of partial conservation of the axial current, which has led to predictions that agree satisfactorily with experiment [4]. We note that in order for (2) to be satisfied it is sufficient to have $\partial L_\mu^\alpha(x,t)/\partial x_\mu$ to transform in accordance with a representation with unity U-spin.

Using the technique developed in [2] together with assumption I, we can easily prove the renormalizability of the vector constant (the Ademollo-Gatto theorem [5]) for particles with arbitrary spin belonging to any SU(3) representation [6]. To this end we must consider a system of commutators (1) between single-particle states and confine ourselves to single-particle intermediate states by virtue of I. A solution of the obtained system comprises the Clebsch-Gordan coefficients of the SU(3) group. The spin independence of the particles follows from the fact that $Q_\alpha^V(t)$ does not change the particle polarization. The meaning of the Ademollo-Gatto theorem consists in the fact that in the given approximation the single-particle states are transformed in accordance with a definite SU(3) representation, in spite of the difference in the corresponding masses.

Starting from the commutator (2) and using the vector current for the operator L_μ^α, we obtain the Gell-Mann - Okubo mass formula, which is linear for baryons and quadratic for bosons, in analogy with [2], but for particles with arbitrary spin.

Taking the commutator (3) between single-particle states belonging to different SU(3) multiplets, we can prove the proportionality of the form factors $\langle a|L_\mu^\alpha|b\rangle$ (if suitably chosen) to the Clebsch-Gordan coefficients of the SU(3) group, at least when $\Delta^2 = 0$, where $\Delta^2 = (P_a - P_b)^2$.

Taking the commutator (2) between different states and knowing from the foregoing how the form factors are renormalized, we obtain the following mass formulas.

1. For arbitrary choice of the operator L_μ^α, satisfying condition II, we prove the validity of the Gell-Mann - Okubo formula within the multiplet; unlike the preceding, however, we cannot prove that only a linear formula holds for baryons and a quadratic one for bosons. The reason is that in using a vector current for L_μ^α we neglect the terms containing the product of two nondiagonal elements $\langle a|Q_\alpha^V(t)|b\rangle$ [2].

2. By considering the commutator (2) between different multiplets of SU(3) and using the fact that there exists at least one operator L_μ^α transforming one multiplet into another - axial current, we obtain universality of the mass (mass-squared) difference of particles with quantum numbers K and π for octets of mesons with arbitrary spins. This relation was derived for the 0^- and 1^- octets in [7] from other considerations; $K^2 - \pi = 0.226$ GeV^2 for 0^-, $K^{*2} - \rho^2 = 0.212$ GeV^2 for 1^-, and $K^2_{1405} - A^2_{2\ 1324} = 0.224$ GeV^2 for 2^+. The agreement can be regarded as satisfactory. We note that the foregoing implies universality of the parameters of the Okubo formula for the mass differences of bosons, in agreement with the existence of a heavier third quark [8].

Among the baryons we have a relation between the masses of the octet and decuplet with arbitrary spins, $\Xi_\delta^0 - \Sigma_\gamma^0 = \Xi^0 - \Sigma^0$. $\Xi_\delta^0 - \Sigma_\delta^0 = 147$ MeV for the $3/2^+$ decuplet, $\Xi^0 - \Sigma^0 = 125$ MeV for the $1/2^+$ octet, and $\Xi_{1820} - \Sigma_{1660} = 160$ MeV for the $3/2^-$ octet. This connection was obtained in SU(6) between the $1/2^+$ octet and the $3/2^+$ decuplet under certain additional assumptions [9].

Between the baryon octets we obtain the relation

$$a_F(\Xi^0 - P - \Xi_\gamma^0 + P_\gamma) + a_D(\Sigma^0 - \lambda - \Sigma_\gamma^0 + \lambda_\gamma) = 0. \qquad (4)$$

If the representation $\underline{27}$ for baryons exists, then each bracket in (4) vanishes, corresponding to universality of the parameters of the Okubo formula for the baryon mass differences. If this is not the case, then (4) is an equation for the F/D ratio, provided (2) and (3) are satisfied only by an axial current.

We have thus shown that some observed regularities in the mass spectrum can be explained on the basis of a scheme of weakly broken symmetry, without resorting to fundamental hypotheses (existence of quarks, higher symmetries).

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CONCERNING THE DECAY $K_L^0 \to \pi^+ + \pi^-$

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1. It was observed in [1] and also in the succeeding experiments [2-4] that, contrary to the theoretical prediction based on the assumed validity of CP invariance,

$$\frac{K_L^0 \to \pi^+ + \pi^-}{K_L^0 \to \text{(all charged channels)}} \simeq 2 \times 10^{-3}. \qquad (1)$$

It is interesting to note that in no other experiments for checking CP invariance (in particular, in lepton decays of K^0 [5,6]) were any effects observed of the same order as in the fundamental experiment [1] (a good review of the entire K_L^0 problem is contained in [6]).

We propose here one more idea for explaining the experiments of [1-4], based on the specific character of the creation and decay of K^0, which has no analog for any other elementary particle. The nub of the idea is the following question: Do the properties of unstable particles depend on the method of preparation [7-11] or not?

2. We assume, as usual, that strong interactions create a K^0 which is a coherent mixture of K_1^0 and K_2^0:

$$K^0 = 1/\sqrt{2}\,(K_1^0 + K_2^0), \qquad (2)$$

where K_1^0 and K_2^0 are the eigenstates of the combined-parity operator $CP|K_1^0\rangle = +1\cdot|K_1^0\rangle$, $CP|K_2^0\rangle = -1\cdot|K_2^0\rangle$ which decay (and are also produced in place of K^0) as a result of weak interaction. The energy (mass) distribution of K^0, on the basis of (2), is

$$\rho_{K^0}(E) = \frac{1}{4\pi}\,\frac{\Gamma_1}{(E - E_{K_1^0})^2 + \Gamma_1^2/4} + \frac{1}{4\pi}\,\frac{\Gamma_2}{(E - E_{K_2^0})^2 + \Gamma_2^2/4}\,, \qquad (3)$$