i.e., of non-coherence.

PHASE TRANSITION IN SUPERCONDUCTORS OF SMALL SIZE

V. V. Shmidt
A. A. Baikov Metallurgy Institute
Submitted 20 December 1965
ZhETF Pis'ma 3, No. 3, 141-145, 1 February 1966

Large fluctuations in the ordering parameter, which lead to the "smearing" of the second-order phase transition over a certain temperature interval, should occur near the critical temperature T_c. In the case of a superconductor, these are fluctuations of the "effective wave function" of the Ginzburg-Landau theory [1] or, from the point of view of the microscopic theory, fluctuations of the energy gap $2\Delta(T)$.

The question of density fluctuations of the superconducting electrons that should be present when $T > T_c$ and, as it were, anticipate the superconducting transition, was raised already in [2], where the possible effect of the fluctuations about T_c on certain characteristics of the superconductor were discussed.

It is known, however, that in pure homogeneous superconductors the transition to the normal state is very abrupt. The temperature region ΔT near T_c, in which $\overline{|\Delta\Psi|^2}$ is comparable with $|\Psi|^2$, amounts in accord with [3] to $\Delta T \sim 10^{-14} - 10^{-15}$ °K. A similar estimate of the width ΔT of the region of the logarithmic singularity in the temperature variation of the specific heat near the transition point was arrived at by the authors of [4].

So narrow a phase-transition region is attributed to the fact that the fluctuations of Ψ in pure superconductors are strongly "suppressed." Actually the fluctuation probability increases exponentially with decreasing volume, but to realize the fluctuation in a small volume of a large superconductor it is necessary to have a large gradient of Ψ. This reduces greatly the fluctuation probability, which decreases exponentially with increasing $(\Delta\Psi)^2$. Thus, fluctuations with small gradients should encompass large volumes of matter and have therefore low probability, while fluctuations in small volumes are accompanied by large gradients and have therefore likewise low probability.

The purpose of this investigation was to show that there exists an object for which the fluctuations of Ψ may turn out to be appreciable. These are superconducting particles of a size small compared with ξ_0 (the dimension of the Cooer pair). If, starting from physical considerations, we cut off the Fourier-integral expansion at a wave number ξ_0^{-1}, then it is easy to show that when $|\Psi|^2$ is calculated for such particles it is possible to neglect the gradient term, and the expression for the free-energy density in the absence of a magnetic field takes the form [1]

$$\Delta F = F_{s0} - F_{n0} = \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 \qquad (1)$$

Since this expression has been written out without the kinetic-energy term $(\hbar^2/2m)|\Delta\Psi|^2$, the states Ψ and Ψ^* are physically indistinguishable and therefore we shall regard Ψ as real.

The probability of fluctuation of the quantity Ψ will, in accord with the Boltzmann principle, be $w \sim \exp(-\Delta F V/kT)$, where V is the volume of the particle. Then

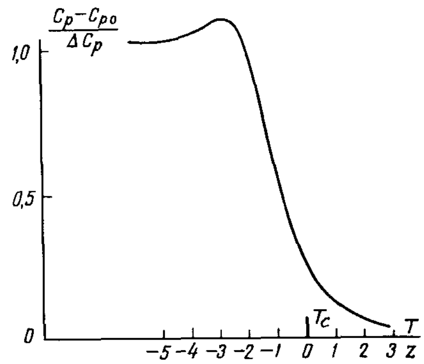
$$\overline{\Psi^2} = \frac{1}{2}\sqrt{\frac{kT}{\beta V}\frac{\mathcal{D}_{-3/2}(z)}{\mathcal{D}_{-1/2}(z)}}, \quad z = \alpha\sqrt{\frac{V}{\beta k T}}, \quad \alpha = a(T - T_c). \tag{2}$$

Here $\mathcal{D}_p(z)$ is the parabolic cylinder function.

Large fluctuations of the quantity Ψ lead to the appearance of an anomalous variation of the specific heat near T_c. If we calculate the specific heat by the method proposed in [5], we obtain the following expression

$$C_p = C_{p_0} + \frac{1}{2}\Delta C_p\left(1 - \frac{1}{2}\frac{\mathcal{D}^2_{-3/2}(z)}{\mathcal{D}^2_{-1/2}(z)} - z\frac{\mathcal{D}_{-3/2}(z)}{\mathcal{D}_{-1/2}(z)}\right), \tag{3}$$

where C_{p_0} is the specific heat in the normal state. A plot of $C_p(T)$ given by (3) is shown in the figure.



From (3) and from the figure it follows that the temperature region ΔT near T_c, where the fluctuations give rise to an anomalous behavior of the specific heat, is determined by the relation $|z| \sim 1$, i.e., $\Delta T \sim T_c(k/V\Delta C_p)^{1/2}$, where $\Delta C_p = a^2 T_c/\beta$ is the jump in specific heat in the transition point. For lead particles with dimension $\sim 10^{-6}$ cm we have

$$\Delta T \sim 0.7°K$$

($T_c = 7.2°K$, $\Delta C_p \sim 10^4$ erg/deg-cm^3).

It follows from (3) that only the width of the transition region depends on the volume of the particle. The asymptotic behavior at large $|z|$ will be as follows: $C_p = C_{p_0} + \Delta C_p/2z^2$ when $z \gg 1$ and $C_p = C_{p_0} + \Delta C_p(1 + z^2/2)$ when $z \ll -1$.

The results obtained should be treated with caution, since the expansion (1) is apparently not valid in the direct vicinity of T_c, where the fluctuations of Ψ are large, i.e., in the region $|z| < 1$. However, the asymptotic estimates for $|z| > 1$ are perfectly reliable.

Similar results are obtained for finely-dispersed superconductors and in investigations of phase transitions based on the magnetic field. It is known from experiment [6] that the magnetic moment of colloidal superconducting particles does not vanish at the critical magnetic field, but attenuates gradually as the field increases above critical. This phenomenon has hitherto been explained as due to the differences in the dimensions of the colloidal particles, which results in a certain scatter in the values of the critical magnetic field. It is clear now that besides this effect there should exist also the smearing of the transition by the occurence of fluctuations, a fact that must be allowed for in the interpretation of the corresponding experiments.

In conclusion we note the following circumstance.

The theory of the phase transition in a superconductor [7], based on the use of the BCS model Hamiltonian, does not lead to singularities of the specific heat at the transition point, but to a finite discontinuity. A more accurate account of the interaction between the electrons in the superconductor should lead to the appearance of a singularity in the specific heat at the transition point [4]. In a bulky superconductor, however, as indicated above, the temperature interval near T_c, where this singularity becomes noticeable, is so small that an experimental investigation of this phenomenon is impossible. The calculation presented shows that for finely-dispersed superconductors this temperature interval increases by many orders of magnitude and can reach values of the order of one degree. An experimental investigation of this phenomenon would contribute to a refinement of our ideas concerning the interaction between electrons in a superconductor.

I am deeply grateful to V. L. Ginzburg and D. A. Kirzhnits for help and an evaluation of the work.

[1] V. L. Ginzburg, JETP 20, 1064 (1950).

[2] V. L. Ginzburg, UFN 46, 348 (1952).

[3] V. L. Ginzburg, FTT 2, 2031 (1960), Soviet Phys. Solid State 2, 1924 (1961).

[4] E. G. Batyev, A. Z. Patashinskii, and V. L. Pokrovskii, JETP 46, 2093 (1964), Soviet Phys. JETP 19, 1412 (1964).

[5] A. P. Levanyuk, FTT 5, 1776 (1963), Soviet Phys. Solid State 5, 1294 (1964).

[6] D. Shoenberg, Proc. Roy. Soc. 175A, 49 (1940).

[7] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, Metody kvantovoi teorii polya v statisticheskoy fizike (Methods of Quantum Field Theory in Statistical Physics), p. 394, Fizmatgiz, 1962.

1) We expand Ψ in a Fourier integral, $\Psi = \int \Psi_q \exp(i\vec{q}\cdot\vec{z})d\vec{q}$. The wave vector q corresponds to the pair momentum $\hbar q$. The condition for pair stability is $\hbar q v_0 \leq 2\Delta_0$, where $2\Delta_0$ is the energy gap at T = 0 and v_0 is the electron velocity on the Fermi surface. Consequently $q \leq 2\Delta_0/\hbar v_0 \sim \xi_0^{-1}$.

EFFECT OF ELECTRIC FIELD ON TRANSPORT PHENOMENA IN POLAR GASES WITH NONSPHERICAL MOLECULES

L. L. Gorelik and V. V. Sinitsyn
Submitted 25 December 1965
ZhETF Pis'ma 3, No. 3, 145-149, 1 February 1966

It is known that the viscosity and thermal conductivity of gases with nonspherical molecules changes in a magnetic field [1-6]. The effect is attributed to the fact that precession of the magnetic moments of these molecules in the magnetic field increases the effective cross section for their collision, and consequently decreases the transport coefficients [7-9]. At constant temperature this effect is a single-valued function of the ratio of the magnetic field to the pressure. It would be natural to expect an analogous effect of the electric field and the transport coefficients of polar gases with nonspherical molecules. We have therefore undertaken investigations of the influence of an electric field on the thermal conductivity of