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## CONCERNING TWO-NUCLEON RESONANCES

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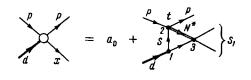
Belletini et al. [1] have noted in their experimental paper an anomalous behavior of the differential cross section of the reaction

$$p + d \rightarrow p + x \tag{1}$$

as a function of the missing mass  $m_x$ . In the region of  $m_x$  = (2.33 ± 0.01) GeV there is a clearly pronounced maximum with width  $\Gamma$  = 250 MeV. Under the experimental conditions the square of the 4-momentum transfer varied in the range  $10^{-3} < t(\text{GeV/c})^2 < 10^{-1}$ .

We propose in this note a mechanism for this intensification, based on the notion that the incident proton interacts with one of the nucleons of the deuteron, forming an isobar N\* which is subsequently scattered inelastically by another nucleon. It is natural to choose as N\* the isobar with mass  $m_{N*}$  = (1.4 ± 0.01) GeV ( $\Gamma$  = 200 MeV), which was observed by the same group in the reaction  $p + p \rightarrow p + x$  and was more pronounced than the remaining isobars.

The amplitude of the reaction (1) in the region of interest to us can be represented as a sum of diagrams (see Fig. 1) which contribute to the amplitude of the reaction (1).



Here and throughout we use the following notation: S - energy squared in the c.m.s. of the factor  $S_1$  and deuteron,  $S_1$  - energy squared in the c.m.s. of the particles emitted at the vertex 3. If we denote by  $a_{\triangle}$ S - energy squared in the c.m.s. of the incident proton the amplitude of the triangular diagram and by  $\mathbf{a}_{\!\scriptscriptstyle \bigcap}$  the amplitude corresponding to all other terms, then the dif-

Fig. 1 ferential cross section can be expressed in the form  $d\sigma = \left|a_0 + a_1\right|^2 d\tau$ , where  $d\tau$  is the phase volume of the final particles of reaction (1).

In the given experiment (with small t and  $m_{_Y} = \sqrt{S_1} \simeq m_{_N} + m_{_{N\!-\!N\!-\!N}}$ ) we can assume that the

energy release at vertices 1, 2, and 3 is small compared with the virtual-particle masses in the triangular diagram, and consequently the entire diagram can be treated as nonrelativistic. The explicit expression for the amplitude  $a_{\wedge}$  with constant vertices is [2]

$$a_{\Delta} = \frac{i}{8\pi} \frac{m_{N} m_{N*}}{\sqrt{m_{N} m_{x}}} g_{1}g_{2}g_{3} \frac{1}{t} ln \frac{\sqrt{t} + \sqrt{t_{\Delta}}}{\sqrt{t} - \sqrt{t_{\Delta}}}, \qquad (2)$$

where

$$t_{\Delta} = 2m_{N} m_{N*} \left[ \sqrt{\epsilon_{d}/2m_{N}} + \sqrt{\epsilon_{m_{X}}/(m_{N} + m_{N*})} \right]^{2}$$

 $\epsilon_{\rm d}$  is the deuteron binding energy,  $\epsilon_{\rm m_X} = {\rm m_N} + {\rm m_{N^*}} - {\rm m_X}$ , and  ${\rm g_1}$ ,  ${\rm g_2}$ ,  ${\rm g_3}$  are the amplitudes of the virtual reactions at vertices 1, 2, 3. It follows from (2) that  ${\rm a_{\Delta}}$  has on the physical sheet a moving complex singularity in t at t =  ${\rm t_{\Delta}(m_X)}$ . The position of the singularity can also be obtained from general considerations without directly calculating the triangular diagram [3,4].

If we separate the terms linear in  $\mathbf{a}_{\triangle}$  from the expression for the differential cross section of reaction (1), viz.,

$$d\sigma = \left[ \left| a_0 \right|^2 + 4 \left( \operatorname{Re} a_0 \cdot \operatorname{Re} a_{\wedge} + \operatorname{Im} a_0 \cdot \operatorname{Im} a_{\wedge} \right) + \left| a_{\wedge} \right|^2 \right] d\tau, \tag{3}$$

then the calculations show that sufficiently good agreement with experiment can be attained by including only the interference terms, in particular Re  $a_0$ ·Re  $a_{\Delta}$ . It is assumed throughout that the amplitude  $a_0$  is a slowly varying function of  $m_{\chi}$ . The phase volume of the three nucleons in the final state, integrated over the emission angles of the particles in vertex 3, is of the form  $d\tau_3 = 2pp'(4\pi)^{-4}(m_{\chi}S)^{-1}dSdn$ , where p and p' are the momenta in the c.m.s. of the particles emitted in vertex 3 and in the c.m.s. of all the final particles of the reaction, and  $d\vec{n}$  is the solid angle for proton emission in the c.m.s. The final expression for the experimentally measured differential cross section is

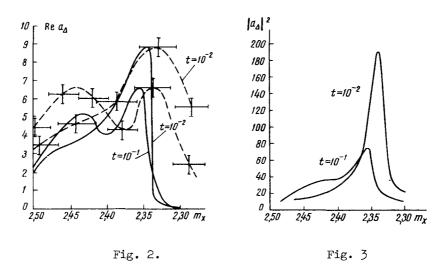
$$\frac{d^2\sigma}{d\eta dS} = \left[ \left| a_0 \right|^2 + 4 \left( \operatorname{Re} a_0 \cdot \operatorname{Re} a_\Delta + \operatorname{Im} a_0 \cdot \operatorname{Im} a_\Delta \right) + \left| a_\Delta \right|^2 \right] f(m_x, S), \tag{4}$$

where  $f(m_X,S) = 2pp!(4\pi)^{-4}(m_XS)^{-1}$ . There is practically no change in  $f(m_X,S)$  in the region of  $m_X \simeq m_N + m_{NX}$ . In the case when one more particle (pion) is emitted in vertex 3,  $f(m_X,S)$  remains likewise practically constant.

Figures 2 and 3 show Re  $a_{\Delta}$  and  $|a_{\Delta}|^2$  under the assumption that  $g_3 > 0$ . For comparison we show the experimental data from [1].

The dashed curve shows the experimental data for  $t = 10^{-1}$  and  $10^{-2}$  (GeV/c)<sup>2</sup>, taken from [1]. The theoretical curves are normalized at the maxima.

The square of the amplitude of the triangular diagram  $|a_{\triangle}|^2$  also has a maximum in this region of  $m_X$ , but the form of the exponential curve corresponds better to allowance for Re  $a_{\triangle}$ ·Re  $a_{\triangle}$  in expression (4) for the differential cross section of reaction (1).



The author is deeply grateful to I. S. Shapiro for continuous interest in the work and a valuable discussion of the results.

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## CORRECTION

to article "Lepton Decay of Vector Mesons" by V. M. Shekhter (JETP Letters 2, 486 (1965), transl. p. 302)

- 1. No account is taken in the article of the fact that the transition matrix element  $\langle \gamma \rho^0 \rangle$  is determined by the renormalized  $\rho$ -meson polarization operator  $\Pi$ , whereas the mass of  $\rho^0$  includes also a renormalization constant  $Z_{\rho}$ , viz.,  $m_{\rho}^2 = m_0^2 + Z_{\rho}^1\Pi$  ( $m_0^1$  is the bare mass of the  $\rho$  meson). In the language of Feyman diagrams this means that  $m_{\rho}^2$  is determined only by the irreducible self-energy diagrams, and not by all diagrams. If the relation for  $m_{\rho}^2$  is written in the form  $m_{\rho}^2 = \Pi + \left[m_0^2 + (Z_{\rho}^1 1)\Pi\right]$ , then the hypothesis advanced in the paper is equivalent to the assumption that  $\Pi \approx m_{\rho}^2 \gg m_{\rho}^2 + Z_{\rho}^1 1)\Pi$ . The quantity in the right side of this inequality is in all cases finite, by virtue of renormalizability, and can be regarded as "the finite part of the bare mass." The author is grateful to A. A. Polyakov for calling his attention to the presence of the factor  $Z_{\rho}^1$  in the self-energy diagram, and to A. A. Ansel'm for a discussion.
- 2. The value of the relative probability of the decay  $\phi^0 \rightarrow e^+e^-$  listed in the table corresponds to the hypothetical (not experimental) cross section  $\sigma(\pi^- + p \rightarrow n + \phi) = 50 \ \mu b$ .