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CONTRIBUTION TO THE THEORY OF QUANTUM OSCILLATIONS OF SURFACE IMPEDANCE

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It is well known that diamagnetic Ladau-level quantization of the electron energies at temperatures that are low compared with the Fermi degeneracy temperature leads to quantum oscillations of both thermodynamic and kinetic quantities. In sufficiently strong magnetic fields ($\hbar\Omega > 2\pi^2 kT$, $\Omega\tau > 1$) these oscillations are not exponentially small. (This is the only case discussed here.) In the static case, when the electrostatic and electromagnetic effects separate, there is also separation of the oscillations of the thermodynamic quantity - the magnetic moment (Shubnikov - van Alphen effect) - and of the kinetic quantity - the resistance (Shubnikov - de Haas effect). The relative amplitude of the quantum oscillations of the conductivity is then proportional to the "quasiclassic parameter" $\kappa = (e\hbar H_0/cS)^{\frac{1}{2}}$ (H_0 = constant magnetic field, S = area of extremal section of the Fermi surface) and is of the order $\Delta \sim \Delta\sigma_{qu}/\sigma_{cl} \sim \kappa p$, where $p \sim \sigma_{cl}^{anom}/\sigma_{cl}^{main}$. The factor p is the result of the fact that the quantum oscillations are connected with anomalously small groups of electrons, whereas the classical conductivity is due to the main electron groups, which are absent only from the metals of the bismuth group. The amplitude of the oscillations of the magnetic susceptibility $\chi_{ik} \equiv \partial\mathcal{M}^i_{st}/\partial n^k_{st}$ ($\mathcal{M}$ - magnetic moment, an expression for which is given in [3]) is determined at low temperature both by the "quasiclassic parameter" and by the "relativistic parameter" $\rho = v_0/c$ (v_0 = Fermi velocity of the electrons) and is of the order of $\chi_{st} \sim \rho^2\kappa^3$. In an alternating electromagnetic field (and of course in a quantizing constant magnetic field) it is meaningful to speak of oscillations of a single quantity, the total surface impedance. These oscillations are connected both with nonrelativistic oscillations of the conduction current and with the relativistic oscillations of the magnetic moment, which are essentially of a different order of magnitude. It is of interest to estimate the contribution of each of these quantities to the impedance and to determine the relative magnitude of the quantum oscillations of the impedance in different cases.

This is simplest to do in the low-frequency region, when $\omega\tau \ll 1$, so that the system "manages" to follow the frequency (ω = frequency of the alternating field, τ = mean free path time), and $\delta_{eff} \gg r, l$ (r = Larmor radius of the electrons, $l \sim v_0\tau$ = mean free path, δ_{eff} = effective thickness of the skin layer), so that at microscopic distances r or l the fields can be regarded as homogeneous, and all the relations can be regarded as local. In the main approximation it is then possible to use the formulas derived for the static case [1-3], and the magnetic susceptibility can be regarded as static. It is easy to obtain the surface impedance Z from Maxwell's equations, by means of the obvious calculations. It turns out that the relative contribution of γ, compared with the magnetic moment, is of the order of $\gamma \sim \Delta\sigma_{qu}/\sigma_{cl}X$. Estimates show that it is always small, so that the surface-impedance oscillations are determined essentially by the de Haas - van Alphen effect, and the relative amplitude of the impedance oscillations is of the order of $\Delta Z/Z \sim (4\pi X)^{-\frac{1}{2}}$.

It must be emphasized that, owing to the periodic dependence of the quantum oscillating terms on $cS/e\hbar H_0$, a nonlinearity appears in these terms even in a relatively weak alternating field. It is readily seen that an external electromagnetic field amplitude $H_1 \gtrsim H_0(e\hbar H_0/cS)$ is sufficient. This remark pertains, of course, to all frequencies.

In a thin plate of thickness d, where $d \ll \delta_{eff}$ but $d \gg r$, l and the electron level quantization is the same as in a bulk sample [4], the role of δ_{eff} will be played by d, so that when $d \ll \delta_{eff}$ the oscillations of Z are determined by the current-density oscillations (Shubnikov - de Haas effect).

With increasing frequency, the relative role of the oscillation of the magnetic moment decreases. First, the magnetic moment is nonlocally connected with the magnetic field intensity, so that the value of the magnetic moment decreases by a factor δ_{eff}/r when the magnetic field is strictly parallel to the sample surface, and by a factor δ_{eff}/l if it is inclined to the surface. Second, the system cannot follow the variations of the alternating field, $\omega\tau \gg 1$, and this decreases the magnetic moment by another factor $\omega\tau$. At sufficiently high frequency the surface-impedance oscillations are determined by the Shubnikov - de Haas effect (the frequency and the order of $\Delta A/A_0$ can be readily estimated from the consideration given above and from the formulas in [1]). It must also be recognized that in the anomalous skin effect ($\delta_{eff} \ll r$ in a parallel field and $\delta_{eff} \ll l$ in a field inclined to the surface) the case $n_1 = n_2$ (n_1 and n_2 are the numbers of the electrons and holes, respectively) ceases to differ from the case $n_1 \neq n_2$ (for more details see [6]). The corresponding formulas are given in [5,6].

Thus, for a specified plate thickness d, the oscillations of Z exhibit with increasing frequency a transition from the Shubnikov - de Haas effect to the de Haas - van Alphen effect and then back to the Shubnikov - de Haas effect.

To construct a consistent theory for the general case it is necessary to take account first of the fact that we are dealing with a system of free charges in external fields, i.e., with a typical field-theory problem described by Maxwell's equations with ordinary boundary conditions (for the field intensities, and not for the induction). The displacement current must be neglected in these equations, for this would mean going beyond the accuracy employed [7]. The current density (including both the conduction current and the current connected with the inhomogeneous magnetization of the sample) has in the quasiclassical approximation the form (for a derivation see [5], Sec. 2):

$$\vec{j}(\vec{R}) = \frac{e}{2}\,\mathrm{Sp}\{\delta(\vec{R} - \hat{\vec{R}}')(\hat{f}\hat{\vec{v}} + \hat{\vec{v}}\hat{f})\},$$

where $\hat{f}$ is the density matrix of the system under consideration. To determine $\hat{f}$ it is necessary to solve a quantum kinetic equation similar to that solved in [5]. In the general case, however, when the relaxation time is introduced, it is important to choose correctly the function that causes the collision term to vanish. It is clear that when $\omega\tau \ll 1$ the principal approximation is $f_0(\epsilon - \gamma)$, where f_0 is the Fermi function and γ is the chemical potential in the specified external field. When $\omega\tau \gg 1$, and the system does not have time to become attuned, the main approximation is $f_0(\epsilon - \gamma_0)$, where γ_0 is the chemical potential in the equilibrium case.

Calculations confirm all the foregoing estimates. The derivation of exact formulas and comparison with experiment will be the subject of a separate communication.

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SIZE EFFECT ON "INEFFECTIVE" ELECTRONS OF OPEN SECTIONS

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Several recent papers are devoted to investigations of radio-frequency size effects. These effects constitute essentially singularities, periodic in the magnetic field, in the surface impedance of plane-parallel metallic plates. These singularities are connected with anomalous penetration of the high-frequency field into the metal. As shown in [1,2], these anomalies can be explained by considering the interaction of the electrons that move in the interior of the metal with the spatial harmonics of the electromagnetic field, which has a sharp inhomogeneity near the metal surface. If the electron moves on trajectories on which there are points with zero velocity in the interior of the metal, the electrons interact most effectively with those field-spectrum harmonics whose wavelength is an integer fraction of the length of the extremal displacement of the electrons in the interior of the metal. In this case a series of harmonics is separated from the continuous spectrum of the electromagnetic field, and their superposition leads to the appearance of periodic field peaks at distances (from the surface) that are multiples of the extremal displacement of the electrons. The plot of the impedance of a plane-parallel plate against the magnetic field then exhibits sharp singularities, which are priodic in the direct field. If there are no points with zero velocity on the orbit in the interior of the metal, the electrons interact only with the field-