APPLICATION OF SU(3) SYMMETRY TO MULTIPLE MESON PRODUCTION

E. I. Daibog
Joint Institute of Nuclear Research
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Watson proved [1], using isotopic invariance considerations, that the following relation is satisfied for multiple production of mesons in a collision between a nucleon and a nucleus containing an equal number of neutrons and protons:

$$\frac{\bar{N}^0}{\bar{N}^+ + \bar{N}^-} = \frac{1}{2},$$

where $\bar{N}^0$, $\bar{N}^+$, and $\bar{N}^-$ are the average numbers of the π^0, π^+, and π^- mesons [1]. In experiments in the high energy region (10^{10} - 10^{12} eV) this ratio is close to unity [2]. It is possible that the quantity actually measured is the ratio $(\bar{N}^0 + \bar{N}^{\eta_0})/(\bar{N}^+ + \bar{N}^-)$ where $\bar{N}^{\eta_0}$ is the average number of η_0 mesons. The purpose of this paper is to calculate the latter ratio with the aid of SU(3) symmetry.

Let us write out the octet of pseudoscalar mesons (the same holds for vector mesons) in the form of a column:

$$\begin{pmatrix} K^+ \\ K^0 \\ \pi^+ \\ \pi^0 \\ \pi^- \\ \eta^0 \\ \bar{K}^0 \\ K^0 \end{pmatrix}$$

We construct the projection operators n_i^α, which act on the wave function of the i-th meson and have eigenvalues 1 if the meson occupies position α in the column, and 0 in the remaining cases. Their form is:

$$n^{K^+} = \tfrac{1}{2}(YI_+I_- + (YI_+I_-)^2),$$

$$n^{K^0} = \tfrac{1}{2}(YI_-I_+ + (YI_-I_+)^2),$$

$$n^{K^0} = \tfrac{1}{2}((YI_+I_-)^2 - YI_+I_-),$$

$$n^{K^-} = \tfrac{1}{2}((YI_-I_+)^2 - YI_-I_+),$$

$$n^{\pi^+} = \tfrac{1}{2}(I_+I_- - I_+I_-L_+L_- - I_-I_+K_+K_- + \tfrac{1}{2}(YI_-I_+ + (YI_-I_+)^2) - \tfrac{1}{2}((YI_+I_-)^2 - YI_+I_-)),$$

$$n^{\pi^0} = \tfrac{1}{2}(I_+I_-L_+L_- + I_-I_+K_+K_- - \tfrac{1}{2}(YI_+I_- + (YI_+I_-)^2) - \tfrac{1}{2}(YI_-I_+ + (YI_-I_+)^2)),$$

$$N^{\pi^-} = \tfrac{1}{2}(I_-I_+ - I_+I_-L_+L_- - I_-I_+K_+K_- + \tfrac{1}{2}(YI_+I_- + (YI_+I_-)^2 - \tfrac{1}{2}((YI_-I_+)^2 - YI_-I_+)),$$

$$n^{\eta^0} = \tfrac{1}{3}(K_+K_- + L_+L_-) - \tfrac{1}{2}(I_+I_- + I_-I_+) + \tfrac{1}{2}(I_+I_-L_+L_- + I_-I_+K_+K_-) - \tfrac{7}{4}(YI_+I_- + YI_-I_+)$$
$$- \tfrac{5}{4}((YI_+I_-)^2 + (YI_-I_+)^2)),$$

where Y, I_+I_-, K_+K_-, and L_+L_- are the group generators [3]. It is easy to obtain with the aid of these operators an extension of the Watson theorem to include SU(3). This extension consists in the fact that the mean values of n_i^α over some set of quantum states φ_ν^μ are the same for all α:

$$\bar{n}_i^\alpha = \sum_\nu (\varphi_\nu^\mu, n_i^\alpha \varphi_\nu^\mu) = \frac{1}{8}.$$

Thus, if all the ν-states of the multiplet μ are realized with equal probability during the particle collison, then the number of produced mesons is equal, for if only a total of N mesons are produced, then by defining

$$N^\alpha = \sum_i n_i^\alpha,$$

we get

$$\bar{N}^{K^+} = \bar{N}^{K^0} = \ldots = \bar{N}^{K^-} = \frac{N}{8}.$$

This, however, is not the formulation of the problem in the experimental case. Let us consider a case when a nucleon collides with a nucleus containing an equal number of neutrons and protons. The collisions in the nucleus are assumed paired. The (II_3Y) states realized are (002) from {10*} and (122) and (102) from {27}. Let us obtain the mean values of the quantities $\sum_\nu N_\nu$ of interest to us over these states, with the aid of the Clebsch-Gordan coefficients for SU(3). The result is as follows:

$$\frac{(\varphi^{10^*}|\pi^+ + \pi^-|\varphi^{10^*})}{(\varphi^{10^*}|\pi^0 + \eta^0|\varphi^{10^*})} = 0.76,$$

$$\frac{(\varphi^{27}|\pi^+ + \pi^-|\varphi^{27})}{(\varphi^{27}|\pi^0 + \eta^0|\varphi^{27})} = 1.02.$$

In other words:

$$0.76 < \frac{\bar{N}(\pi^+ + \pi^-)}{\bar{N}(\pi^0 + \eta^0)} < 1.02.$$

It follows from the foregoing that for physically realizable states we can obtain inequalities which make it possible to estimate the ratio of the charged and neutral components.

The entire analysis is based on the assumption that the SU(3) symmetry is exact. We note that an estimate of the phase volumes shows that in the ultrarelativistic region the phase volume is practically independent of the masses (this takes place already for multiple production processes observed with the aid of an accelerator).

[1] K. N. Watson, Phys. Rev. 85, 852 (1952).

[2] V. S. Murzin, Izv. AN SSSR ser. fiz. 28, 1790 (1964), transl. Bull. Acad. Sci. Phys. Ser. p. 1680.

[3] J. J. de Swart, Revs. Modern Phys. 35, 916 (1963).