

STATE OF LIQUID HELIUM IN THE VICINITY OF THE λ LINE

L. V. Kiknadze, Yu. G. Mamaladze, and O. D. Cheishvili
 Physics Institute, Georgian Academy of Sciences
 Submitted 31 January 1966
 ZhETF Pis'ma 3, No. 8, 305-309, 15 April 1966

Let liquid helium completely fill a vessel, in which the pressure P_0 at the lid and the temperature T are such that at a certain depth z_λ the pressure corresponds to the λ line ($P_0 + \rho g z_\lambda = P_\lambda(T)$)¹⁾. One might think that the diagram of state calls for the helium to be in the state of He II in the upper part of the vessel ($0 \leq z \leq z_\lambda$) and in the state of He I in the lower part ($z_\lambda \leq z \leq H$). The purpose of this article is to prove the assertion that liquid helium should actually be either superfluid throughout or normal throughout.

This assertion follows from an examination of the equation of the phenomenological superfluidity theory of Ginzburg and Pitaevskii [1]:

$$\frac{\hbar^2}{2m} \Delta \Psi + \alpha \Psi - \beta |\Psi|^2 \Psi = 0, \quad (1)$$

where m is the mass of the helium atom, $\Psi = |\Psi| \exp(i\varphi)$ is the wave function defining the density ρ_s and the velocity \vec{v}_s of the superfluid component ($\rho_s = m |\Psi|^2$, $\vec{v}_s = \hbar \nabla \varphi / m$), and α and β are coefficients that depend on the temperature and on the pressure. In our case $\nabla \varphi = 0$, so that we can put $\varphi = 0$. In addition, neglecting the dependence of the helium density ρ on the pressure and temperature, we can express the coefficient α in the form

$$\alpha = \alpha'(T_{\lambda 1} - T) \left[1 + \left(\frac{dT}{dP} \right)_\lambda \frac{P - P_{\lambda 1}}{T_{\lambda 1} - T} \right] = \alpha'(T_{\lambda 1} - T) \frac{z_\lambda - z}{a_\lambda} \quad \text{if } 0 \leq z \leq z_\lambda, \quad (2)$$

$$\alpha = 0 \quad \text{if } z_\lambda \leq z \leq H,$$

where $\alpha' \approx 4.5 \times 10^{-17}$ erg/deg [1], and the parameter a_λ is determined by

$$a_\lambda = \frac{P_\lambda(T) - P_{\lambda 1}}{\rho g} \approx 7 \times 10^5 (T_{\lambda 1} - T) \text{ cm}. \quad (3)$$

When the temperature changes from $T_{\lambda 2}$ to $T_{\lambda 1}$ this parameter changes from a quantity of the order of 2×10^5 to zero. We note that when $P_\lambda(T) \gg P_1$ and $P_\lambda(T) \gg P_0$ the quantities z_λ and a_λ are close to each other. On the other hand, if $P_\lambda(T) \gtrsim P_0 \gg P_{\lambda 1}$ then $z_\lambda \ll a_\lambda$.

We introduce the notation $\alpha_1 = \alpha'(T_{\lambda 1} - T)$, $a_0^2 = \hbar^2 / (2m\alpha_1)$, $x = (a_0^2 a_\lambda)^{-1/3} (z_\lambda - z)^2$, and $f = (a_\lambda / a_0)^{1/3} (\beta / \alpha_1)^{1/2} |\Psi|$. Then in the upper part $0 \leq z \leq z_\lambda$ of the vessel (the "superfluid" region) Eq. (1) takes the form

$$\frac{d^2 f}{dx^2} + x f - f^3 = 0 \quad \text{if } 0 \leq x \leq h_s, \quad (1a)$$

where $h_s = (a_0^2 a_\lambda)^{-1/3} z_\lambda$. In the lower part $z_\lambda \leq z \leq H$ of the vessel ("normal" region) Eq. (1) becomes

$$\frac{d^2 f}{dx^2} - f^3 = 0 \quad \text{if } -h_n \leq x \leq 0, \quad (1b)$$

where $h_n = (a_0^2 a_\lambda)^{-1/3} (H - z_\lambda)$.

The first integrals of these equations are

$$\left(\frac{df}{dx}\right)^2 - \frac{1}{2}f^4 + x f^2 + \int_x^{h_s} f^2 dx = \left(\frac{df}{dx}\right)_{h_s}^2 \quad \text{if } 0 \leq x \leq h_s, \quad (4a)$$

$$\left(\frac{df}{dx}\right)^2 - \frac{1}{2}f^4 = \left(\frac{df}{dx}\right)_{-h_n}^2 \quad \text{if } -h_n \leq x \leq 0, \quad (4b)$$

where allowance is made for the boundary conditions $f(h_s) = f(-h_n) = 0$, and the derivatives (df/dx) at the points $x = h_s$ and $x = -h_n$ are for the time being undetermined constants³⁾. As in many other cases considered in [1-4], Eq. (4b) establishes the dependence of f on x with the aid of an elliptic integral of the first kind. However, unlike all these cases, the first integral (4a) has no algebraic expression of (df/dx) in terms of f , thus making the general solution of the problem difficult.

Nonetheless, we can assert that in general there exists a nonzero solution of (1), defined in the entire vessel, that is to say, there exists a function f made up of solutions of (1a) and (1b) which are smoothly joined together at the point $x = 0$. This means in turn that superfluidity is possible also in the "normal" region. (The function $|\Psi|$ increases in this case from zero at $z = 0$, goes through a maximum somewhere in the "superfluid" region, and decreases monotonically to zero at $z = H$.) However, under certain critical conditions such a function can vanish identically, and then the liquid remains normal even in the "superfluid" region.

In view of the already mentioned mathematical difficulties, we shall merely confirm the consideration just advanced by means of an approximate calculation, which is valid in the particular case of an "infinitely" deep "normal" region, covered by a "thin" superfluid layer $h_n \gg h_s$ or $H \gg z_\lambda$. More accurately, we assume that h_n is so large that $(df/dx)_{-h_n}$ can be set equal to zero. The meaning of the statement concerning the smallness of h_s will be explained later.

Under these conditions, Eq. (4b) simplifies and the solution of Eq. (1b) for an "infinite" "normal" region is written in the form

$$f = -\left(\frac{x}{\sqrt{2}} \frac{1}{f(0)}\right)^{-1} \quad \text{if } -\infty < x \leq 0. \quad (5b)$$

We determine the solution of (1a), bypassing the first integral (4a), by expansion in powers of x , satisfying the conditions of smooth joining with expression (5b):

$$f = f(0) \left\{ 1 + \frac{1}{\sqrt{2}} f(0) x + \frac{1}{2} f^2(0) x^2 + \frac{1}{6} \left(\frac{3}{\sqrt{2}} f^3(0) - 1 \right) x^3 + \frac{1}{4} f(0) \left(f^3(0) - \frac{1}{\sqrt{2}} \right) x^4 \right.$$

$$+ \frac{1}{20} f^2(0) \left(\frac{7}{2\sqrt{2}} f^3(0) - 1 \right) x^5 + \dots \} \quad (5a)$$

The undetermined constant $f(0)$ must be found from the boundary condition $f(h_s) = 0$.

The last condition determines the critical thickness of the "superfluid" region h_{sc} , below which (when $h_s \leq h_{sc}$) Eqs. (1a) and (1b) have only a trivial zero solution. Indeed, using formula (5a) to determine $f(0)$ by means of the equation $f(h_s) = 0$, we can readily verify that it has no nonzero solutions ($f(0) \equiv 0$) if

$$1 + \sum_{k=1}^{\infty} (-1)^k \frac{h_s^{3k}}{3k(3k-1)(3k-3)(3k-4)\dots 3 \cdot 2} = 0, \quad (6)$$

which yields $h_{sc} \approx 2$. (This is indeed the measure of the "thinness" of the "superfluid" layer, for when $h_s \gtrsim h_{sc}$ the series (5a) converges rapidly.)

Reverting to dimensional notation, we obtain the following estimate for the critical thickness of the "superfluid" layer of the "infinitely" deep "normal" region $z_{\lambda c} \approx 2(a_0^2 a_{\lambda})^{1/3} \approx 2.2 \times 10^{-3}$ cm.

The shift of the λ point, due to the external pressure, is described by the following formula

$$\delta T_{\lambda} = \frac{P_0 - P_1 + \rho g z_{\lambda c}}{-\left(\frac{dP}{dT}\right)_{\lambda}} \quad (7)$$

When $P_0 - P_1 \gg \rho g z_{\lambda c}$ formula (7) practically coincides with the equation of the λ line, but when P_0 is small it describes the additional shift of the λ point, connected with the phenomena considered in this note. In particular, the shift of the λ point at saturated vapor pressure is equal to

$$\delta T_{\lambda s} = \frac{\rho g z_{\lambda c}}{\left(\frac{dP}{dT}\right)_s - \left(\frac{dP}{dT}\right)_{\lambda}} \approx 3 \times 10^{-9} \text{ } ^\circ\text{K}, \quad (8)$$

where the derivative $(dP/dT)_s$ is taken along the curve $P = P_s(T)$ at $T = T_{\lambda 1}$. The exceedingly small value of $\delta T_{\lambda s}$ calculated in this manner denotes that the large volume of liquid helium in equilibrium with its own vapor will be in the superfluid state at practically any temperature $T < T_{\lambda 1}$, even if its greater part satisfies the condition $P > P_{\lambda}(T)$.

- [1] V. L. Ginzburg and L. P. Pitaevskii, JETP 34, 1240 (1958), Soviet Phys. JETP 7, 858 (1958).
- [2] Yu. G. Mamaladze and O. D. Cheishvili, JETP Letters 2, 123 (1965), transl. p. 76.
- [3] O. D. Cheishvili and Yu. G. Mamaladze, Phys. Lett. 18, 278 (1965).
- [4] Yu. G. Mamaladze and O. D. Cheishvili, JETP 50, 169 (1966), Soviet Phys. JETP 23, in press.

1) The z axis is directed downward, $0 \leq z \leq H$, where H is the height of the vessel. It is understood throughout that $0 < z_{\lambda} < H$ and also that $P_s(T) \leq P_0 < P_{\lambda}(T)$, $P_s(T) < P_{\lambda 1}$, $P_{\lambda 1} < P_{\lambda}(T) < P_{\lambda 2}$, and $T_{\lambda 2} < T < T_{\lambda 1}$, where ρ_s is the saturated vapor pressure, while $(T_{\lambda 1}$ and $P_{\lambda 1})$ and $(T_{\lambda 2}$ and $P_{\lambda 2})$ are the points on the ends of the λ line ($T_{\lambda 1}$ is the λ point at saturated vapor pressure). Assuming the λ line to be straight, we have:

$$(dP/dT)_\lambda = -(P_{\lambda 2} - P_{\lambda 1}) / (T_{\lambda 1} - T_{\lambda 2}) \approx -100 \text{ at/deg.}$$

2) An important factor is that the theory contains a temperature-independent characteristic dimension $(a_0^2 a_\lambda)^{1/3} \approx 1.1 \times 10^{-3} \text{ cm}$. It is implied throughout that the transverse dimensions of the vessel are much larger than all the three characteristic dimensions a_0 , a_λ , and $(a_0^2 a_\lambda)^{1/3}$ of the theory.

3) We note the interesting equation
$$\int_0^{x_0} f^2 dx = \left(\frac{df}{dx} \right)_{h_s}^2 - \left(\frac{df}{dx} \right)_{-h_n}^2 .$$

INTENSIFICATION OF A SHOCK WAVE BY THE POLYMERIZATION ENERGY AND THE FEASIBILITY OF A POLYMERIZATION DETONATION

I. M. Barkalov, V. I. Gol'danskii, V. L. Tal'roze, and P. A. Yampol'skii
 Institute of Chemical Physics, USSR Academy of Sciences
 Submitted 17 February 1966
 ZhETF Pis'ma 3, No. 8, 309-312, 15 April 1966

The polymerization of several solid monomers by a shock wave was recently observed [1]. Thus, for example, it was noted that passage of a shock wave with amplitude of 50 kbar or higher causes 60% of acryl amide to be converted into a polymer. It is known [2] that the polymerization energy of acryl amide is about 10 - 15 kcal/mole. Thus, the energy released in 60% polymerization of acryl amide is $E_p = 10 \text{ kcal/mole}$. It is of interest to compare this quantity with the energy E_s obtained by the substance as a result of compression by the shock wave. The change in the internal energy of one gram of matter compressed by a shock wave, without allowance for the energy of the chemical processes, is equal to $E_s = P(V_0 - V)/2$, where V_0 is the specific volume of the substance prior to compression, and V is the specific volume of the substance compressed to a pressure P . To obtain the dependence of the specific volume of the substance on the applied pressure it is necessary to determine the shock adiabat of the investigated substance.

To determine these quantities in acryl amide we use the values obtained for Plexiglas and polystyrene, which have the same initial density and approximately equal compression coefficients. We assume that the shock adiabat for the acryl amide differs little from the shock adiabats of Plexiglas [3] and polystyrene [4], which are very close to each other at $P < 100 \text{ kbar}$. Then at a pressure $P = 5 \times 10^{10} \text{ dyne/cm}^2$ the relative compression is $V/V_0 = 0.74$. From this we get $E_s \approx 10 \text{ kcal/mole}$ for acryl amide with $V_0 = 0.85 \text{ cm}^3/\text{g}$. As is well known [4], at pressures lower than $10^{11} \text{ dyne/cm}^2$ the thermal component of the internal energy of the compressed substance does not exceed half the total energy. Therefore heating of the substance by the shock wave amounts to approximately 200°C . On the other hand, the shock of polymerization (assuming that it occurs during the passage of the shock wave) heats the substance by 400°C ¹). The total temperature rise of the compressed substance is therefore not less than 600°C .

If the resultant mixture of monomer and polymer were to be cooled by thermal conductivi-