EFFECT OF RETARDED INTERACTION ON THE EXCITON SPECTRUM IN ONE-DIMENSIONAL AND TWO-DIMENSIONAL CRYSTALS

V. M. Agranovich and O. A. Dubovskii
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In interactions between excitons and photons in three-dimensional crystals, the wave vector is fully conserved, accurate to the integer reciprocal-lattice vector. Therefore allowance for retarded interaction in such crystals leads to new states - normal electromagnetic waves, which are undamped if anharmonicity is not taken into account (see, e.g. [1], Sec. 8, and also [2-4]). In this case, naturally, the dielectric tensor remains real [5].

Unlike three-dimensional crystals, allowance for retarded interaction in one- and two-dimensional periodic structures leads to several singularities in the exciton spectra. The reason is that when excitons and photons interact in one- and two-dimensional crystals the quantities conserved, accurate to the integer reciprocal-lattice vector, are either only one or only two projections of the wave vector.

Let us consider this question in greater detail [1)]. Using a Coulomb gauge for the vector potential and following [4], we represent the Hamiltonian of the system excitons + transverse-photon field in the form

$$\hat{H} = \hat{H}_{Coul} + \hat{H}_{\perp} + \hat{H}_{int}, \tag{1}$$

where

$$\hat{H}_{Coul} = \sum_{\vec{k}\mu} E_{\mu}(\vec{k})\hat{B}^{+}_{\mu}(\vec{k})B_{\mu}(\vec{k}) \tag{2}$$

is the Hamiltonian of the excitons, corresponding to full account of the Coulomb interaction,

$$\hat{H}_{\perp} = \sum_{\vec{q}j} \hbar qc\ \hat{a}^{+}_{j}(\vec{q})\hat{a}_{j}(\vec{q}) \tag{3}$$

is the Hamiltonian of the transverse-photon field, and

$$\hat{H}_{int} = -\sum_{\vec{n}} \frac{e}{mc}\ \hat{\vec{A}}(\vec{n})\hat{\vec{I}}(\vec{n}) \tag{4}$$

is the main term in the exciton-photon interaction operator. In (2) and (3) $E_{\mu}(\vec{k})$ is the energy of an exciton with wave vector in band μ, $\hat{B}^{+}_{\mu}(\vec{k})$, $\hat{B}_{\mu}(\vec{k})$ and $\hat{a}^{+}_{j}(\vec{q})$, $\hat{a}_{j}(\vec{q})$ are the Bose creation and annihilation amplitudes of the excitons and photons, respectively, $\vec{q}$ is the wave vector of the photon, and $j = 1, 2$ its polarizations. In (4) $\hat{\vec{A}}(\vec{n})$ is the vector-potential operator and $\hat{\vec{I}}(\vec{n})$ the electron-momentum operator of the molecule $\vec{n}$. If the photon wave vector

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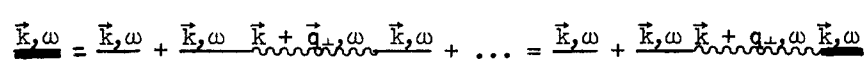
$\vec{q}$ is represented as the sum $\vec{q} = \vec{q}_{\parallel} + \vec{q}_{\perp}$, where $(\vec{q}_{\parallel} \cdot \vec{q}_{\perp}) = 0$, and the component $\vec{q}_{\perp}$ is perpendicular to the exciton wave vectors, then the operator (4) can be reduced in the second-quantization representation (see also [4]) to the form (we disregard here inessential processes such as umklapp):

$$\hat{H}_{int} = \sum_{j\mu\vec{q}} T_{j\mu}(\vec{q}) \hat{a}_j(\vec{q}) [\hat{B}_\mu(-\vec{q}_{\parallel}) - \hat{B}^+_\mu(\vec{q}_{\parallel})] + \text{H.c.}, \tag{5}$$

$$T_{j\mu}(\vec{q}) = i(2\pi N/V\hbar cq)^{\frac{1}{2}} E_\mu(\vec{q}_{\parallel})(\vec{l}_j(\vec{q})\vec{P}_\mu(\vec{q}_{\parallel})), \tag{5a}$$

where N is the number of molecules in the cyclicity volume of the crystal ($N = N_1$ and $N = N_1N_2$ for one-dimensional and planar crystals, respectively), V the cyclicity volume for photons, $V = N_1N_2N_3d^3$, d is the lattice constant, $\vec{l}_j(\vec{q})$ is the unit vector of the photon polarization $(j\vec{q})$, and $\vec{P}_\mu(\vec{q}_{\parallel})$ is the matrix element of the dipole moment operator of the transition from the ground state to the exciton state $(\mu\vec{q}_{\parallel})$. Relation (5a) is valid only if $q < 1/d$. If $q > 1/d$, then $T_{j\mu}(\vec{q}) \to 0$ (see [4]). We therefore confine ourselves below in the summation to the region $q \leq q_0 \simeq 1/d$. The operator (5) leads to processes in which the exciton is converted into photons and vice versa, with conservation of the wave-vector component $\vec{q}_{\parallel}$. If we denote by a solid line the Green's function of the photon, then, obviously the perturbation-theory series for the exciton Green's function is



(6)

Here the dots denote the vertex part in the zeroth approximation. Solving this equation with respect to the exciton group function, we find that its poles are given by the equation

$$\epsilon^2 - E^2_\mu(\vec{k}) = \sum_{j\vec{q}_\perp} 4\hbar cq\, E_\mu(\vec{k}) \left| T_{j\mu}(\vec{q}) \right|^2 (\epsilon^2 - \hbar^2c^2q^2)^{-1}. \tag{7}$$

For a linear chain this equation takes the form

$$E^2_\mu(\vec{k}) - \epsilon^2 = A(\vec{k}) \int_0^{q_0} \frac{dz}{z - a} \left[1 - \frac{k^2\cos^2\theta + \frac{z}{2}\sin^2\theta}{k^2 + z} \right] \tag{8}$$

where

$$A(\vec{k}) = \frac{8\pi^2 E^3_\mu(\vec{k})}{d\hbar^3c^3} \left| \vec{P}_\mu(\vec{k}) \right|^2, \quad a \equiv \frac{\epsilon^2 - \hbar^2c^2k^2}{\hbar^2c^2}. \tag{9}$$

Since A is proportional to $\left|\vec{P}(\vec{k})\right|^2$, it is clear that the role of the retardation can be significant only for sufficiently intense dipole transitions. If, for example, $E_\mu \simeq 5$ eV, $\left|\vec{P}_\mu\right| \simeq e \times 10^{-8}$ cm, and $d \simeq 5 \times 10^{-8}$ cm, then $A \simeq 1\ (\text{eV})^2$.

The solutions of (8) are, generally speaking, complex: $\epsilon = \epsilon' + i\epsilon''$. In the spectral region where $|\epsilon''| < \epsilon'$ we have

$$\frac{1}{z - a} = \mathcal{P}\frac{1}{z - a} + i\pi\delta(z - a).$$

Neglecting the attenuation in the first approximation and taking the integral in (8) in

the sense of the principal value, we obtain for $k \ll q_0$ and $|a| \ll q_0$ the following equation for $\epsilon'(\vec{k})$:

$$E_\mu^2(\vec{k}) - (\epsilon') = \frac{A}{2}\left\{(1 + \cos^2\theta)\ln\left|\frac{\hbar^2c^2q_0^2}{(\epsilon')^2 - \hbar^2c^2k^2}\right| + \frac{1 - 3\cos^2\theta}{(\epsilon')^2}\hbar^2c^2k^2\ln\left|\frac{k^2}{a}\right|\right\} \quad (10)$$

Here θ is the angle between the vector $\vec{P}_\mu(\vec{k})$ and the axis of the chain.

A solution of this equation is shown schematically in Fig. 1. We find that

$$\epsilon''(\vec{k}) = \frac{A}{2\epsilon'}\left[1 - \frac{k^2\cos\theta + (a/2)\sin^2\theta}{k^2 + a}\right] \quad (11)$$

when $\epsilon'(\vec{k}) > \hbar kc$ and $\epsilon''(\vec{k}) = 0$ when $\epsilon'(\vec{k}) < \hbar kc$. Thus, the states of the lower branch of Fig. 1 do not attenuate, whereas the states of the upper branch have an appreciable radiation width. The cross on Fig. 1 denotes arbitrarily the limit of the spectrum: here $|\epsilon''| \simeq \epsilon'$. $\epsilon'' \cong A/2\epsilon'$ when $k \simeq 0$, and if $A = 1\ (eV)^2$ and $\epsilon' = 5$ eV, then $\epsilon'' \cong 0.1$ eV.

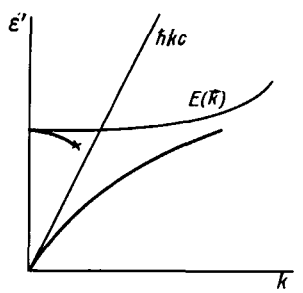


Fig. 1

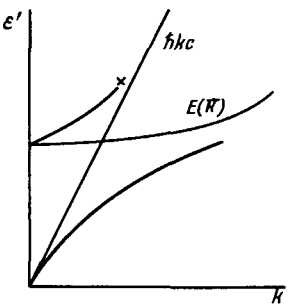


Fig. 2

We can consider similarly the case of a planar crystal. Assuming for simplicity that the vector $\vec{P}_\mu(0)$ is perpendicular to the plane of the crystal, we find that when $k \ll q_0$ the equation for ϵ' has the following form

$$E_\mu^2(\vec{k}) - (\epsilon')^2 = B\frac{\hbar^2k^2c^2}{(\epsilon')^2}\left\{\int_0^{q_0}\frac{dq_\perp}{q_\perp^2 - a} - \frac{\pi}{2k}\right\}, \quad (12)$$

where

$$B = 8E_\mu^3(\vec{k})\left|\vec{P}_\mu(0)\right|^2/d^2\hbar^2c^2.$$

At the same time

$$|\epsilon''| = B\hbar^2k^2c^2/2(\epsilon'(\vec{k}))^3(a)^{\frac{1}{2}}, \quad (13)$$

if $\epsilon'(\vec{k}) > \hbar kc$ (upper branch of the spectrum), and $\epsilon''(\vec{k}) = 0$ if $\epsilon'(\vec{k}) < \hbar kc$ (lower branch of the spectrum). A plot of $\epsilon'(\vec{k})$ is shown in Fig. 2. For the upper branch of the spectrum we have with high accuracy

$$\epsilon'(\vec{k}) \cong E_\mu(\vec{k})\left[1 + \frac{k}{\tilde{k}}\right], \quad (14)$$

where $\tilde{k} = 4E_\mu^4/\pi B\hbar^2c^2$. The excitons of this branch have a relatively large group velocity, $v \cong E_\mu(0)/\hbar\tilde{k} \simeq 10^7 - 10^8$ cm/sec and should interact weakly with the lattice vibrations. The

cross on Fig. 2 denotes the limit of the spectrum. Here the elementary-excitation attenuation due to the possibility of emitting a photon becomes too large ($|\epsilon''| \sim \epsilon'$).

The foregoing singularities of the spectra can occur if the uncertainty of the wave vector, connected with the finite size of the crystal, is smaller than the wave vector $k \sim E_\mu/\hbar c$, in the vicinity of which the retardation is significant. It follows therefore that the retardation effects can take place if the dimensions of the crystals exceed the wavelength $\lambda = hc/E_\mu$. The excitons should have in such crystals a lifetime $\sim 10^{-13} - 10^{-15}$ sec, which can be discerned from the luminescence damping time, from the absorption and emission line shapes, etc. There should be practically no Stokes shift of the lines in such systems.

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[1)] The radiation width of the excited states of nuclei in one-dimensional and planar structures are considered in several papers [6,7].

CONTRIBUTION TO THE QUANTUM THEORY OF ELECTRIC CONDUCTIVITY OF SEMICONDUCTORS WITH NON-STANDARD BAND

B. M. Askerov and F. M. Gashimzade
Institute of Physics, Azerbaidzhan Academy of Sciences
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We generalize here the theory developed by Adams and Holstein [1] to include the case of an isotropic but nonstandard band (such as the conduction band of InSb, InAs, etc.), and discuss the influence of the spin splitting of the Landau levels on the oscillations of the transverse magnetoresistance in n-InSb. The dispersion law for such semiconductors was obtained by Kane [2], and also by Bowers and Yafet [3] for the case when a magnetic field is present.

To solve the equation of motion for the density matrix by the method of Adams and Holstein [1], we have determined the electron spectrum in crossed electric and magnetic fields