Another interesting fact is that an increase in the depth of modulation is accompanied not only by a narrowing of the high-frequency spectrum, but also by a shift of the frequency of the maximum oscillation amplitude to the modulating frequency. When $\alpha \simeq 0.15$ the maxima of the oscillations coincide. As seen from Fig. 2d, the spectrum in the 50 kcs - 30 Mcs region is then fully suppressed. In the 0 - 50 kcs region the amplitude drops by a factor 2 - 3.

Simultaneously with the change that modulation produces in the spectrum, a 30% decrease is observed in the ion current (in individual cases up to 40%). The plasma column diameter decreases when the low-frequency oscillations are stopped. This decrease in diameter is apparently due to the decrease in the anomalous diffusion due to the low-frequency oscillations.

The experimental results indicate that prior modulation of the beam makes it possible to suppress not only the high-frequency oscillations over a wide range of frequencies, but also the low-frequency oscillations excited by the two-stream instability [4,5]. We have shown that the low-frequency oscillations are produced by the high-frequency ones.

For a conclusive answer to the question whether low-frequency oscillations are the result of nonlinear interaction between high-frequency waves or to static potential wells caused by the high-frequency fields additional research is necessary.

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A NEW RESONANCE CONNECTED WITH MUTUAL DRAGGING OF ELECTRONS AND PHONONS

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This paper is devoted to an investigation of the effect of the mutual dragging of electrons and phonons on the propagation of electromagnetic waves in semimetals and degenerate semiconductors situated in an external magnetic field.

Assuming the electric field to be weak and negelecting spatial dispersion, we seek the distribution functions f and N for the electrons in the form

$$f = [\exp(\frac{\epsilon(p) - \epsilon_0}{T}) + 1]^{-1} + (\vec{\chi}(p), \frac{\vec{p}}{p}), \quad N = [\exp(\frac{\hbar\omega(q)}{T} - 1]^{-1} + (\vec{\psi}(q), \frac{\vec{q}}{q}). \qquad (1)$$

Here $\vec{p}$ is the electron quasimomentum, m its effective mass, $\epsilon(p) = p^2/2m$ its energy, ϵ_0 the Fermi energy, T the temperature, $\omega(q)$ the phonon frequency, and q the phonon quasimomentum. The additions to the equilibrium functions f_0 and N_0 are assumed small.

In analogy with [1], we can obtain from the system of kinetic equations for f and N the following system of equations for $\vec{\chi}$ and $\vec{\psi}$:

$$\frac{d\vec{\chi}}{dt} + \frac{e}{mc}[\vec{H} \times \vec{\chi}] + \nu_e\vec{\chi} + \frac{\pi}{\epsilon}\frac{df_0}{d\epsilon}\int_0^{2p}\vec{\psi}(q)q^2\hbar\omega(q)W(q)dq = -\frac{e\vec{E}p}{m}\frac{df_0}{d\epsilon},$$

$$\frac{\partial\vec{\psi}}{\partial t} + \nu_{ph}\vec{\psi} - \frac{T4\pi mW(q)}{\hbar\omega(q)}\int_{q/2}^{\infty}\vec{\chi}(p)dp = 0, \tag{2}$$

where $\vec{H}$ is the external magnetic field, $\vec{E}$ the alternating electric field, e the electron charge, W(q) the probability of electron-phonon collision with a change of momentum by q, $\nu_e = \nu_{e-i} + \nu_{e-ph}$, ν_{e-i} and ν_{e-ph} the frequencies of electron collisions with impurities and phonons, respectively, $\nu_{ph} = \nu_{ph-e} + \nu_{ph-ph} + \nu_{ph-i} + \nu_{ph-b}$; ν_{ph-e}, ν_{ph-ph}, ν_{ph-i}, and ν_{ph-b} are the frequencies of phonon collisions with electrons, phonons, impurities, and sample boundaries. The formulas for ν_{e-ph} and ν_{ph-e} are

$$\nu_{e-ph} = \frac{\pi T}{p\epsilon}\int_0^{2p}\frac{q^3W(q)}{\hbar\omega(q)}dq, \quad \nu_{ph-e} = \frac{4\pi m^2W(q)\hbar\omega(q)}{q[\exp(\frac{\epsilon(q/2) - \epsilon_0}{T}) + 1]}. \tag{3}$$

It is assumed in the derivation of (2) that $\hbar\omega(q)/T \ll 1$, i.e., the scattering of the electrons by the phonons is elastic. If the electron gas is fully degenerate $(df_0/d\epsilon = -\delta(\epsilon - \epsilon_0))$ and the electromagnetic field is monochromatic with frequency ω $(\vec{E} = \vec{E}_0 \exp[-i\omega t])$, then it is convenient to seek $\vec{\chi}$ in the form

$$\vec{\chi} = \vec{\kappa}(\epsilon_0)\delta(\epsilon - \epsilon_0)\exp(-i\omega t). \tag{4}$$

We then obtain from (2) an easily-solved algebraic equation for $\vec{\kappa}$. Calculating the current $\vec{j}$ with the aid of the electron distribution function, we get:

$$j = \frac{ie^2N\{\vec{E} - \frac{i\omega_H}{\Omega}[\vec{E} \times \vec{h}] - (\frac{\omega_H}{\Omega})^2\vec{h}(\vec{E}\cdot\vec{h})\}}{m(\Omega^2 - \omega_H^2)}, \tag{5}$$

where $\vec{h} = \vec{H}/H$, $\omega_H = |e|H/mc$ is the Larmor frequency, N is the carrier density, and Ω is given by

$$\Omega = i\left(\nu_e - \frac{4\pi^2m^2T}{\epsilon_0p_0}\int_0^{2p_0}\frac{q^2W^2(q)\nu_{ph}(q)}{\omega^2 + \nu_{ph}^2(q)}\right) + \omega\left(1 + \frac{4\pi^2m^2T}{\epsilon_0p_0}\int_0^{2p_0}\frac{q^2W^2(q)}{\omega^2 + \nu_{ph}^2(q)}dq\right), \tag{6}$$

p_0 is the limiting Fermi momentum. It is easily shown that if the following inequalities are satisfied

$$\frac{\nu_{ph}(2p_0)\nu_{ph-e}(2p_0)}{\omega^2} \ll 1, \quad \frac{\nu_{ph-e}(2p_0)\nu_{e-ph}(2p_0)}{\omega^2} \ll 1, \tag{7}$$

then $\Omega = \omega + i\nu_e$.

In this case formula (5) takes the same form as in the absence of dragging, i.e., at sufficiently high frequencies the dragging does not affect the propagation of the electromagnetic waves. If the conditions of strong dragging $\nu_{ph-e} \gg \nu_{ph-ph} + \nu_{ph-i} + \nu_{ph-b}$ are satisfied, and the field frequency is so small that the inequality $\omega^2/\nu^2_{ph-e} \ll 1$ holds, then the expression for Ω is

$$\Omega = i\left(\nu_{e-i} + \frac{\omega^2 T}{\pi m^3 p_0^3}\int_0^{2p_0}\frac{q^5}{(\hbar\omega(q))^3 W(q)}dq + \frac{2T}{mp_0^3}\int_0^{2p_0}\frac{q^4(\nu_{ph-ph} + \nu_{ph-i} + \nu_{ph-b})}{(\hbar\omega(q))^2}dq\right)$$
$$+ \omega\frac{2T}{mp_0^3}\int_0^{2p_0}\frac{q^4}{(\hbar\omega(q))^2}dq. \tag{8}$$

To proceed further we must make more precise the phonon dispersion law and the dependence of $W(q)$ on q. We consider first acoustic phonons, for which $\hbar\omega(q) = sq$ (s is the speed of sound). Simple calculations yield for Ω the formula

$$\Omega = \frac{16}{3}\frac{T}{ms^2}(i\nu' + \omega), \tag{9}$$

where the order of magnitude of ν' is estimated from the relation

$$\nu' \sim \frac{3}{16}\frac{ms^2}{T}\nu_{e-i}(2p_0) + \nu_{ph-ph}(2p_0) + \nu_{ph-b}(2p_0) + \frac{\omega^2}{\nu_{ph-e}(2p_0)}. \tag{10}$$

Substituting (9) in formula (5) for the current, we get

$$\vec{j} = \frac{3}{16}\frac{ie^2N(\omega - i\nu')s^2}{T}\,\frac{\vec{E} - \dfrac{i\tilde{\omega}_H}{\omega + i\nu'}[\vec{E}\times\vec{h}] - \left(\dfrac{\omega_H}{\omega + i\nu'}\right)^2\vec{h}(\vec{E}\cdot\vec{h})}{(\omega + i\nu')^2 - \tilde{\omega}_H^2}, \tag{11}$$

where

$$\tilde{\omega}_H = \frac{3}{16}\frac{|e|Hs^2}{cT}.$$

The denominator in (11) has a resonant character, the resonance occurring at frequency $\tilde{\omega}_H$. The width of the resonance line will be of the order of ν'. We note that by virtue of the assumption that the electron scattering by the phonons is elastic we have $(16/3)(T/ms^2) \gg 1$, and consequently $\omega_H \gg \tilde{\omega}_H$. Thus, the new resonance occurs at a markedly lower frequency than ordinary cyclotron resonance. The range of resonant frequence $\tilde{\omega}_H$ is defined, in accord with the foregoing, by the inequality

$$\nu' \ll \tilde{\omega}_H \ll \nu_{ph-e}. \tag{12}$$

The left-side inequality is necessary to make the line sufficiently narrow. If we assume that the semiconductor or semimetal is sufficiently pure, so that scattering by impurities can be neglected, then $\nu' \sim s/L$, where L is the minimum dimension of the sample. A formula for ν_{ph-e} is given in [1]. Numerical estimates for bismuth show that for $T \sim 10^{-15}$ erg (10°K) and $L \sim 0.1$ cm, $\tilde{\omega}$ lies in the interval between 10^6 and 10^{10} cps. Conditions (12) can be realized in experiments.

A similar calculation for optical phonons under strong-dragging conditions leads to

$$\tilde{\omega}_H = \frac{5}{64(3\pi^2)^{2/3}} \frac{|e|H\omega_0^2}{cTN^{2/3}}$$

(ω_0 is the end-point frequency of the optical phonons). It is apparently a much more complicated matter to observe optical phonons than acoustic ones. We note that the predicted resonance can be readily distinguished from cyclotron resonance, owing to the specific temperature dependence of the resonance frequency.

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E R R A T A

In the article "New Resonance Connected with Mutual Dragging of Electrons and Phonons" by F. G. Bass (Vol. 3, No. 9, p. 233) the factor in formula (9) should be 4/3 and not 16/3, and consequently the factors in (10) and (11) should be 3/4 and not 3/16. In the estimate preceding Eq. (12) it is likewise necessary to replace 16/3 by 4/3.