FILAMENTARY STRUCTURE OF LIGHT BEAMS IN NONLINEAR LIQUIDS

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1. The first observation of the filamentary structure of light in liquids, resulting from self-focusing [1-7], were described by Pilipetskii and Rustamov [4]. Bloembergen and Lallemand [8] relate the anomalously large amplification coefficients in experiments on induced Raman scattering to the same effect 1). However, the theoretical estimates of the self-focusing lengths needed for the formation of filaments, given in [8] (with reference to Kelley [6] 2), are at least one order of magnitude larger than the experimentally observed values. The discrepancy is explained by the fact that Bloembergen and Lallemand start from the notion of the self-focusing of the beam as a whole, confining themselves to a remark concerning the possible role played in the filament formation by small-scale oscillations of intensity in the incident beam.

We present below a theory of filament formation. We show that in a nonlinear dielectric amplitude-phase perturbations of a plane electromagnetic wave bring about its decay 3) into individual beams, which have different self-focusing lengths, depending on the scale of the initial perturbation. There exists in this case a characteristic fastest-focusing scale, determined by the nonlinearity coefficient of the medium, by the intensity, and by the ellipticity coefficient of the wave.

2. The coordinate dependence of a slowly varying amplitude $E_0(\bar{x}, \bar{y}, \bar{z})$ of a linearly polarized monochromatic wave $E = E_0 \exp[i(\omega t - k\bar{z})]$ in an isotropic dielectric with constant $\epsilon_f/\epsilon = 1 + \epsilon'|E|^2$ $(k = \omega\sqrt{\epsilon}/c = 2\pi\sqrt{\epsilon}/\lambda_0)$ is described by the nonlinear equation of transverse diffusion [5,6]

$$\Delta_\perp \epsilon - 2i\frac{\partial \mathcal{E}}{\partial z} + |\mathcal{E}|^2\mathcal{E} = 0. \tag{1}$$

We have introduced in (1) the dimensionless quantities $\mathcal{E} = \sqrt{\epsilon'}E_0$ $(\epsilon' > 0)$, $(x, y, z) = k(\bar{x}, \bar{y}, \bar{z})$.

We can judge the character of decay of the plane wave from the development of small perturbations in it. Putting in (1) $\mathcal{E} = (\mathcal{E}_0 + e)\exp(-j\gamma z)$ $(\gamma = \mathcal{E}_0^2/2$, $\mathcal{E}_0 = \mathrm{const}$ is the amplitude of the unperturbed wave, $|e| \ll \mathcal{E}_0)$ and retaining only terms of first order in $e = e_1 + ie_2$, we get

$$\Delta_\perp e_1 + 2\frac{\partial e_1}{\partial z} + 2\mathcal{E}_0^2 e_1 = 0; \quad \Delta_\perp e_2 - 2\frac{\partial e_1}{\partial z} = 0. \tag{2}$$

For perturbations of the type 4) $e_{1,2} = \mathrm{Re}\, e_{1,2}^0 \exp(-i\vec{\kappa}_\perp \cdot \vec{r} - ihz)$ we get from (2)

$$h^2 = \frac{\kappa_\perp^2}{4}(\kappa_\perp^2 - \kappa_{\perp im}^2); \quad \kappa_{\perp im}^2 = 2\mathcal{E}_0^2 = 16\pi\epsilon' P(cn)^{-1} \equiv \bar{\kappa}_{\perp im}^2, \tag{3}$$

where $n = \sqrt{\epsilon}$ and P is the power flux density in the unperturbed wave.

Perturbations with transverse wave numbers $0 < \kappa_\perp < \kappa_{lim}$ are unstable with respect to z ($h^2 = -\Gamma^2 < 0$), whereas perturbations of smaller scale $\kappa_\perp > \kappa_{lim}$ are stable ($h^2 > 0$). The increment of the unstable perturbations $\Gamma = (2\delta_0^2 - \kappa_\perp^2)^{1/2}\kappa_\perp/2$ has its maximum value $\Gamma_{max} = \delta_0^2/2$ when $\kappa_\perp = \kappa_f = \delta_0$, corresponding to characteristic scales $\Lambda_\perp = \pi/\kappa_f = \pi/\delta_0$ and $\Lambda_\parallel = \Gamma_{max}^{-1}$, or in terms of the dimensional variables

$$\bar{\Lambda}_\perp = \lambda_0 c^{1/2}(32\pi n\epsilon' P)^{-1/2}; \quad \bar{\Lambda}_\parallel = (k\Gamma_{max})^{-1} = 4n\Lambda_\perp^2(\pi\lambda_0)^{-1}. \tag{4}$$

For sufficiently intense perturbations of the incident beam, we can choose $\bar{\Lambda}_\parallel$ to be the characteristic scale of instability development (occurrence of a glowing filament). An estimate of the values of $\bar{\Lambda}_\perp$ and $\bar{\Lambda}_\parallel$ for a beam of 2 mm diameter, 1 MW power, and $\epsilon' = 10^{-11}$ cgs esu (data of Kelley [6]) yields a value $\bar{\Lambda}_\parallel = 9$ cm, which is in good agreement with experiment [8], and a characteristic scale $\bar{\Lambda}_\perp = 180\ \mu$ which naturally exceeds the observed filament dimensions (20 - 80 μ) [8] determined by further self-focusing of the glowing channel.

The power of an individual filament (estimated as the flux of energy through a section of area $\pi\bar{\Lambda}_\perp/4$),

$$W_f \simeq \pi\bar{\Lambda}_\perp^2 P/4 = c\lambda_0^2(128n\epsilon')^{-1}, \tag{5}$$

is equal, apart from a coefficient of order unity, to the critical power of the stationary beam [3]. Since the power W_f does not depend on the power of the initial beam, only the number of filaments increases with increasing power of the initial beam, but not their intensity.

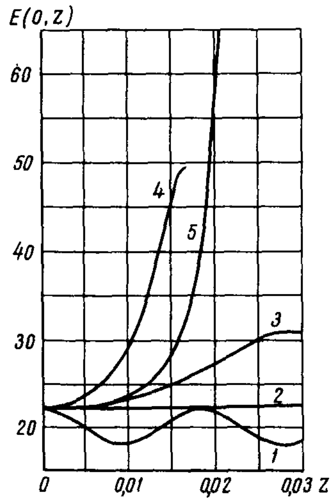
The approximate analysis of the formation of filaments on the basis of the linearized equations in (2) is confirmed also by the results of a numerical solution of the nonlinear equation (1). Figure 1 shows the change in the field amplitude in the plane x = 0 for an initial perturbation of the form $e = 2\cos\kappa_\perp x$ for $\delta_0 = 20$ and different values of $\kappa_\perp$ [5]. The change in the amplitude of the same wave at the point r = 0 under nonperiodic perturbations having a Gaussian profile $e = 2\exp(-r^2/b^2)$ is shown in Fig. 2. In both cases we observe the aforementioned dependence of the amplitude growth rate on the width of the initial-perturbation region.

3. Without dwelling on details, we present the main results pertaining to the occurrence of a filamentary structure in the field of an elliptically polarized wave. The liquid acquires under the influence of such a wave anisotropic properties described by the nonlinear polarization [9]

$$\mathcal{P}_i^{NL} = AE_i(\vec{E}\cdot\vec{E}^*) + \frac{B}{2}E_i^*(\vec{E}\cdot\vec{E}). \tag{6}$$

Here A and B are certain constants. In particular, if the nonlinear properties are due to orientation of anisotropically polarized molecules (the high-frequency Kerr effect), then $A = B/6 = \epsilon\epsilon'/16\pi$, were ϵ' is the nonlinearity coefficient for a linearly polarized wave. For a striction nonlinearity B = 0.

An examination of the development of the perturbations of the amplitudes of right- and left-polarized waves, in which it is expedient to resolve the total field in our case, leads as before to the characteristic equation (3), in which κ_{lim}^2 can now assume two values. In



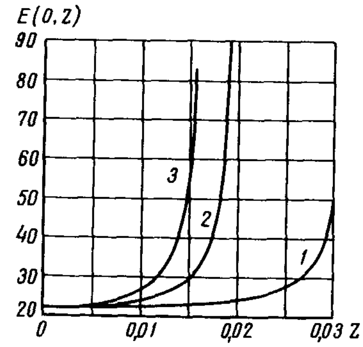


Fig. 2. Variation of amplitude of field $\mathcal{E}(0,z)$ on the beam axis r = 0 for $\mathcal{E}(r,0) = 20 + 2\exp(-r^2/b^2)$. b = 0.5 (1), 0.25 (2), and 0.15 (3).

Fig. 1. Variation of amplitude of the field $\mathcal{E}(0,z)$ in the plane x = 0 for $\mathcal{E}(x,0) = 20 + 2\cos \kappa_\perp x$. $\kappa_\perp^2 = 3\kappa_f^2$ (1), $2\kappa_f^2 = \kappa_{\lim}^2$ (2), $\kappa_f^2(1 + \sqrt{3}/2)$ (3); κ_f^2 (4), and $\kappa_f^2(1 - \sqrt{3}/2)^{-1}$(5).

particular, in the case of the Kerr effect $\kappa_{\lim\,1,2}^2 = \bar{\kappa}_{\lim}^2 F_{1,2}(\beta)$, where

$$F_{1,2}(\beta) = \{1 \pm [1 + 48(1 - \beta^2)(1 + \beta^2)^{-1}]^{1/2}\}$$

is a factor that depends on the ellipticity coefficient β (the ratio of the ellipse axes) of the unperturbed wave. Inasmuch as $F_1 > 0$ and $F_2 < 0$, only one pair of roots ($\pm h_1$) of Eq. (3) corresponds to unstable perturbations when $0 < \kappa_\perp^2 < \kappa_{\lim\,1}^2$. When the ellipticity coefficient increases from 0 (linear polarization) to 1 (circular polarization), the limiting value $\kappa_{\lim\,1}$ changes from $\bar{\kappa}_{\lim}$ to $\bar{\kappa}_{\lim}/2$. Consequently, waves with circular polarization are spatially more stable; this is connected with the decrease of the effective nonlinearity parameter to $\epsilon'/4$, something already noted in the paper of Zel'dovich and Raizer [10]. We note that, owing to the inhomogeneity of the field, the rotation of the polarization ellipse [9] is a function of the transverse coordinates. This, together with the deformation of the initial ellipse by the self-focusing, may be the cause of the depolarization of the radiation after passing through the nonlinear medium [8].

4. A plane wave in a nonlinear medium is unstable not only against small field perturbations but also against small random perturbations of the medium. The field scattered by the inhomogeneities can be taken in some section z > 0 to be the initial perturbation and the analysis can be carried out by the method described above. A more consistent solution of the problem reduces to an investigation of the system (2) (or the corresponding system for elliptically polarized light) with a right-hand side characterizing the scattering of the unperturbed wave by the inhomogeneities. An analysis of the correlation function of the field described by these equations shows that when $z \sim \Lambda_\parallel$ the correlation region of the perturbations is of the order of $\Lambda_\perp$, and the intensity of the scattered field is maximal in the case when the inhomogeneity dimensions are close to $\Lambda_\perp$.

The instability of a plane wave in a nonlinear medium with $\epsilon' > 0$ against perturbations of the field or of the medium may also affect directly the structure of the field radiated by a laser, especially in those cases when the field is acted upon, besides the nonlinearity of the active medium, also by the nonlinearity of other materials placed for various purposes inside the laser cavity (for example, when saturating shutters are used).

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1) Assumptions concerning the role of self-localization in induced scattering of light were made earlier [3].

2) The self-focusing length L_f of an axially-symmetrical beam with a Gaussian amplitude distribution can be estimated approximately with the formula $L_f = ka^2(W_b W_{cr}^{-1} - 1)^{-1/2}$, where a is the beam radius, W_b the beam power, W_{cr} the minimum beam power required for autolocalization. This result follows from Eq. (8) of Talanov's paper [5].

3) The instability of a plane wave in a nonlinear dielectric was noted by R. V. Khokhlov at the 1st All-union Symposium on Nonlinear Optics (Minsk, June 1965).

4) An arbitrary perturbation can be represented as a superposition of such fields.

5) The invariance of Eq. (1) against the substitutions $\mathcal{E} \to \alpha\mathcal{E}$, $\vec{r}_\perp \to \alpha\vec{r}_\perp$, $z \to \alpha^2 z$ always enables us to go over from the given numerical values to physically realizable values of $\mathcal{E}_0$ and $e \ll 1$ [7].

RETARDED NONSTATIONARY RERADIATION OF ELECTROMAGNETIC SIGNALS BY A PARAMETRICALLY REGENERATED FERRITE

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Nonstationary parametric amplification of electromagnetic oscillations of frequency $f/2$ (where f is the pump frequency) were observed for the first time in a magnetized ferrite, the