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STRATIFICATION OF LIGHT BEAMS IN A NONLINEAR MEDIUM AND THE REAL THRESHOLD FOR SELF-FOCUSING

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Stationary self-maintaining radiation channels in a medium with refractive index $n = n_0 + n_2E^2$ were investigated earlier [1-3]. Talanov, Akhmanov, et al. [4,5] investigated the conditions for self-focusing of light beams, but described the latter in terms of spherical waves with variable radius of curvature, thus excluding beforehand the possible stratification and self-channeling (self-trapping) of a part of the beam. Yet the experiments of [6] offer evidence to the contrary.

We consider below the self-focusing of laser beams having a real divergence α. We explain the causes of stratification, and estimate the fraction of the energy that can enter the channel as well as the threshold power P_{thr} of the generator. The latter turns out to be much higher than the critical value P_{cr} [3].

Let a beam of radius R be focused in a medium with a lens of focal length f. The maximum convergence angle of the rays is $\theta_{max} \approx R/fn_0$. The radius of the focusing circle 1) $r_f = f\alpha$ is usually quite large for ruby, and, in contradiction to Talanov, Akhmanov, et al. [4,5], the refraction in the focal region has little effect on θ_{max} (see below). Each element of the focal volume is a source of an elementary cone of rays with half-vertex angle θ_{max}, and rays coming from different elements behind the focal region intersect. However, at distances $x > r_f/\theta_{max}$ along the axis this can be approximately neglected, and the existence of a ray-direction field $\theta(r,x)$ assumed. We assume for simplicity that the angular distribution of the intensity in the beam diverging from the focal region, which duplicates the brightness distribution on the end face of the generator, has axial symmetry, and that the field E decreases monotonically in each beam section with increasing distance from the axis. We assume the process to be stationary.

We start from the equations of quasi-optics for stationary paraxial rays r(x) [4,5] ($\theta_{max} \ll 1$, $n_2E^2 \ll n_0$):

$$\frac{d^2r}{dx^2} = \frac{d\theta}{dx} \approx \nabla_\perp\left(\frac{n_2E^2}{n_0} + \frac{\lambda_0^2}{8\pi^2 n_0^2}\,\frac{\nabla_\perp E}{E}\right). \tag{1}$$

Here λ_0 is the wavelength of the wave in vacuum; $(d\theta/dx) = (\partial\theta/\partial x) + \theta(\partial\theta/\partial r)$. The second equation is written in integral form:

$$\int_0^{r(x)} n_0 \frac{cE^2}{4\pi}\, 2\pi r' dr' = P_1 = \text{const} \tag{2}$$

(the power inside the ray tube is constant).

We insert initial conditions, extrapolating the solution to the focal region, viz.: when $x = 0$

$$r = r_1,\ \theta = \theta_1(r_1),\ E^2 = E_1^2(r_1) \approx E_f^2 = 4P/n_0 c r_f^2,$$

where P is the total beam power. We specify, for estimating purposes, a profile $E^2(r')$ $\sim \exp(-r'^2/r^2)$ inside the tube $r(x)$. This yields

$$\nabla_\perp E^2 = -2E^2/r,\quad \nabla_\perp(\nabla_\perp^2 E/E) = 2/r^3,\quad P_1 = (e - 1)n_0(cE^2/4\pi)\pi r^2.$$

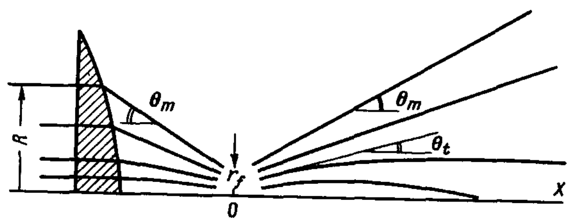
Now the beam equation is

$$\frac{d\theta}{dx} = \frac{1}{2}\frac{d\theta^2}{dr} = -\theta_t^2 \frac{r_1^2}{r^3}(1 - P_{cr}/P_1), \tag{3}$$

where $\theta_t = (2n_2E_f^2/n_0)^{\frac{1}{2}}$ is the total-internal-reflection angle in the focal region; $P_{cr} = (e - 1)c\lambda_0^2/32\pi n_2$ is 2.14 times smaller than the exact value [3]. Let us integrate (3):

$$\theta^2 = A + \theta_t^2(1 - P_{cr}/P_1)(r_1^2/r^2);\quad A = \theta_1^2 - \theta_t^2(1 - P_{cr}/P_1). \tag{4}$$

If $A > 0$ the rays diverge, and as $x \to \infty$ we have $r \to \infty$, $\theta \to \sqrt{A}$, and $r \approx \sqrt{A}x$. If $A < 0$, which is possible only if $P_1 > P_{cr}$, the radius is limited and such beams can become focused in channels (self-channeling). We assume as an estimate that the light intensity in the initial cone $\theta_1 < \theta_{max}$ is constant (the end face of the laser glows uniformly). Then $P_1 = P(\theta_1/\theta_{max})^2$. From the equation $A(\theta_1^2) = 0$ we obtain the limiting value of the initial angle for the rays participating in the self-channeling (see the figure):



Ray diagram (the refraction of the rays at the entrance to the medium is not shown)

$$\theta_1^* = \theta_t(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 - 4P_{cr}\theta_{max}^2/P\theta_t^2})^{\frac{1}{2}} \approx \theta_t. \tag{5}$$

It exists only when $P \geq 4P_{cr}\theta_{max}^2/\theta_t^2(P)$, which determines the threshold power of the entire beam

$$P_{thr} = \frac{4\pi}{\sqrt{e - 1}} \frac{n_0\ r_f\ \theta_{max}}{\lambda_0} P_{cr} = \frac{4\pi}{\sqrt{e - 1}} \frac{R\alpha}{\lambda_0} P_{cr}. \tag{6}$$

The power entering the channel is

$$P_{ch} = P(\theta_1^*/\theta_{max})^2 \approx P(\theta_t/\theta_{max})^2,$$

i.e.,

$$P_{ch}/P = 4P_{cr}P/P_{thr}^2 = (\sqrt{e-1}/\pi)(\lambda_0/R\alpha)P/P_{thr}. \qquad (7)$$

Formulas (6) and (7) are valid if $\theta_t < \theta_{max}$. If $\theta_t > \theta_{max}$, as is the case when $P > P_{max} = (\pi/\sqrt{e-1})(R\alpha/\lambda_0)P_{thr}$, the entire beam may become self-channeling.

We present numerical estimates. When $R = 0.4$ cm, $\alpha = 2.5 \times 10^{-3}$, $\lambda_0 = 7000$ Å, $P_{thr} = 138P_{cr}$, and $P_{max} = 35P_{thr}$. When $P \approx P_{thr}$ we have $P_{ch}/P \approx 3\%$, and $\theta_t/\theta_{max} \approx 0.17$. (If $f = 4$ cm and $n_0 = 1.5$, $\theta_{max} = 0.07$ and $r_f = 10^{-2}$ cm.) We put $n_2 = 2 \times 10^{12}$ absolute units, as is the case for many liquids. Then $P_{cr} = 40$ kW (according to [3] - 87 kW), $P_{thr} = 5.5$ MW, and $P_{max} = 190$ MW. These estimates agree with the results of experiments [6] according to which the power entering the channel at $P \approx 20$ MW is of the order of one per cent.

The formulas for P_{thr}, P_{ch}, and P_{max} are valid also for an unfocused laser beam (under the assumption that the intensity is uniformly distributed within the limits of the divergence angle). In this case r_f is replaced by R and θ_{max} by α. If the divergence is governed by diffraction, as in [8], then $\alpha \sim \lambda_0/R$ and, naturally, $P_{max} \sim P_{thr} \sim P_{ch}$. The beam is then self-focused at a distance $\sim R/\theta_t$ [9]. It must be noted that the question of formation of a channel of constant radius by beams that are self-focused toward the axis still remains unclear in our case, and also in [4,5,9] (the formal solution leads to pulsations). 2)

If we take into account the effect of the nonlinearity of the medium on the ray paths converging from the lens to the focus, we find that a ray having at the lens an inclination θ' acquires at the focus an inclination $\theta'' = \sqrt{\theta'^2 + \theta_t^2}$, i.e., when $P \ll P_{max}$, only rays with $\theta'^2 \lesssim \theta_t^2$ can be deflected noticeably (always towards the axis) when $\theta_t^2 \ll \theta_{max}$.

This, however, should not lead to a noticeable deficit of rays with small initial inclination angles $\theta_1 < \theta_t$, which would greatly increase the threshold power. The points of intersection of the rays in the focal region lie not so much on the axis as in the entire volume. Therefore some rays come close to these points, as they approach the axis, and their slope is increased by refraction, whereas others enter the focal region moving away from the axis, and their slope decreases, thus giving rise to compensation.

Pilipetskii and Rustamov occasionally observed two or three channels in their experiments [6]. This is apparently connected with the fact that the end faces of a real ruby do not glow uniformly, but in "spots." Consequently, the beam diverging from the focus consists of several narrower cones, each of which behaves independently as described above. Sometime filaments are observed in front of the focus of the lens, similar to those usually interpreted as the results of self-channeling. 3) It can be assumed that this is the result of self-channeling of radiation that has experienced backward induced Mandel'shtam-Brillouin scattering.

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1) When $\theta_{max} \ll 1$ the presence of the medium does not change r_f.

2) In considering the channel itself ($d\theta/dx = 0$) it is necessary to make Eqs. (1) and (2) more precise by writing in (1) $\delta\epsilon/2\epsilon_0$ in place of n_2E^2/n_0 and in (2) $\sqrt{\epsilon_0 + \delta\epsilon}$ in place of n_0 ($\epsilon = \epsilon_0 + \delta\epsilon$ is the dielectric constant). In the channel we obtain approximately: $\delta\epsilon(E^2) = \lambda_0^2/4\pi^2r_k^2$ and $P_{ch} = \sqrt{\epsilon_0 + \delta\epsilon}(cE^2/4\pi)\pi r_k^2(e - 1)$, where r_k is the effective radius of the channel. We eliminate the field from these equations by using the interpolation formula $\delta\epsilon = \epsilon_2E^2[1 + \epsilon_2E^2/\delta\epsilon_{max}]^{-1}$, which ensures "saturation" of the polarizability [7] ($\epsilon_2 = 2n_0n_2$, $\delta\epsilon_{max} \sim \epsilon_0$). We then obtain $P_{ch}/P_{cr} = (1 + \delta\epsilon)(1 - \delta\epsilon/\delta\epsilon_{max})^{-1}$, which together with the first of the equations gives the dependence of the radius on the power. When $P_{ch} \approx P_{cr}$ we have $r_k \sim \lambda_0[(P_{ch}/P_{cr}) - 1]^{-\frac{1}{2}}$, as in [3], and when P_{ch} exceeds P_{cr} by several times r_k is of the order of the wavelength and does not depend on P_{ch}.

3) Private communication from N. F. Pilipetskii.

OSCILLATIONS OF THE INTERMEDIATE STATE OF SUPERCONDUCTORS

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In the intermediate state, the volume of a superconductor constitutes a system of alternating layers of normal and superconducting phases. The regular arrangement of the layers corresponds to the minimum of energy, and therefore if the layers are bent by external forces, the system will ultimately return to the unperturbed state. This raises the question of whether this process will constitute weakly damped oscillations. This question is closely connected with the question of the motion of macroscopic filaments of magnetic flux, which was already discussed earlier [1,2].

In this paper we consider small oscillations of the intermediate state, the wavelength of which greatly exceeds the dimensions of the layers of the normal phase.

The motion of the interphase boundaries is accompanied by occurrence of an alternating electromagnetic field in the normal regions. This field is described by the equations

$$\mathrm{curl}\ \vec{H} = \frac{4\pi}{c}\vec{j}, \text{ and } \mathrm{curl}\ \vec{E} = \frac{i\omega}{c}\vec{H}, \qquad (1)$$

where ω is the oscillation frequency.