cidences in the two fast γ detectors. The obtained spectrum shows an intense anomalous maximum near 90 keV, which apparently belongs to the low-energy levels.

The delay factors of the 118- and 256-keV levels relative to the single-particle estimates after Moszkowski, are equal to 9 and 670 respectively.

The latter suggests that the 256-keV level is ℓ-forbidden with multipolarity M1 and level spin $7/2^+$. The lack of additional information on the experimental quantum characteristics of Pm^{151} does not permit an unambiguous discussion of the results.

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SOME CONSEQUENCES OF THE ALGEBRA OF WEAK AND ELECTROMAGNETIC CURRENTS

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A few years ago Nambu [1] proposed that the axial current of β decay is rigorously conserved in the limit when the pion mass vanishes, $m_\pi \to 0$.

It was shown in [1] that the Goldberger-Treiman condition is satisfied exactly in this limit. If we take into account the smallness of the pion mass compared with other hadrons, then it becomes clear why the Goldberger-Treiman condition is apparently satisfied in the real world. In this note, on the basis of Nambu's hypothesis and under the assumption that weak and electromagnetic currents from an SU(3) x SU(3) algebra, we obtain several relations that are valid in the limit as $m_\pi \to 0$. We proceed to derive these relations. Let $j^i_{\alpha 5}(x)$ be the axial current of the i-th isospin component, and $j^i_\alpha(x)$ the vector current of this component. We start from the formula:

$$\partial_\alpha \partial'_\beta \langle B|T\, j^i_{\alpha 5}(x)\, j^k_{\beta 5}(x')|A\rangle = i\epsilon^{ik\ell} \langle B|j^\ell_\gamma(x)|A\rangle\, \partial'_\gamma\, \delta(x - x') \quad (1)$$

$$(\partial_\alpha = \partial/\partial x_\alpha, \quad \partial'_\beta = \partial/\partial x'_\beta).$$

In the derivation (1) we took into account the equalities

$$\partial_\alpha\, j^i_{\alpha 5}(x) = 0; \; [j^i_{05}(x), j^k_{\alpha 5}(x')]_{x_0 = x'_0} = i\epsilon^{ik\ell}\, j^\ell_\alpha(x)\, \delta(\vec{x} - \vec{x}'). \quad (2)$$

If we take the Fourier transforms of (1) with respect to x and x' (corresponding to the momenta q and q') then this equation can be expressed graphically as shown in Fig. 1, where the short dashed lines correspond to the current operators. We now let q and q' approach 0 [1]. In this limit, the main contribution to the left side of (1) are made by diagrams which have pion poles both in q and in q'. If we denote the amplitude of the transition of an axial

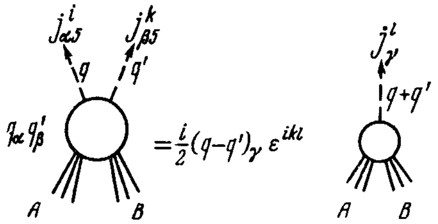

Fig. 1

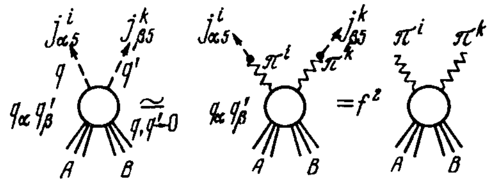

Fig. 2

current into a pion with momentum q by $f \cdot q_\alpha$ and assume that the propagation of the pion corresponds to a propagator $1/q^2$, we obtain in the limit as q, q' → 0 the equation shown in Fig. 2 [2)].

Comparison of Figs. 1 and 2 leads to the formula:

$$f^2 M_{A \to B + \pi^i + \pi^k} = \frac{i}{2} (q - q')_\beta \, \epsilon^{ik\ell} \, M_{A \to B + j^\ell_k} \tag{3}$$

where $M_{A \to B + \pi^i + \pi^k}$ is the sum of diagrams that transform A and B into two pions π^i and π^k with known momenta q and q', while i and k are isotopic indices. $M_{A \to B + j^\ell_\beta}$ is the sum of diagrams that transform A and B into a current j^ℓ_β with momentum q + q'. All the relations of [3] follow from this formula.

To obtain additional relations we use a formula derived by a method similar to that used to derive (1):

$$\begin{aligned} \partial_\alpha \partial'_\beta \langle B | T j^+_{\alpha 5}(x) j^-_{\beta 5}(x') j^{eM}_\gamma(x'') | A \rangle &= \langle B | T j^3_\beta(x) j^{eM}_\gamma(x'') | A \rangle \partial'_\beta \delta(x - x') \\ &+ \langle B | j^3_\gamma(x) | A \rangle \delta(x - x') \delta(x - x''). \end{aligned} \tag{4}$$

Here $j^\pm = (j' \pm i j^2)/\sqrt{2}$ and $j^{eM} = j^3 + j^8/\sqrt{3}$ is the electric current. In addition to (2), we used the formula:

$$[j^+_{05}(x), j^{eM}_\beta(x')]_{x_0 = x'_0} = \delta(\vec{x} - \vec{x}') j^+_{\beta 5}(x). \tag{5}$$

If we separate in the left-hand side of (4) the pole diagrams with respect to the π meson, we obtain

$$f^2 M^\alpha_{A \to B + \pi^+ - \pi^- + \gamma} = \tfrac{1}{2}(q - q')_\beta \, M^{\alpha\beta}_{A \to B + \gamma + \gamma'} + M^\alpha_{A \to B + \gamma'}. \tag{7}$$

Here M are the Feynman amplitudes, γ the photon, γ' the "isovector photon," and α and β their polarizations. The most interesting formulas (7) are obtained if we take $|A\rangle = |\eta^0\rangle$ and $|B\rangle = |0\rangle$:

$$f^2 M^\alpha(\eta^0 \to \pi^+ \pi^- \gamma) = \tfrac{1}{2}(q - q')_\beta \, M^{\alpha\beta}(\eta^0 \to \gamma + \gamma') \tag{8}$$

(the second term in (7) is forbidden by virtue of C-parity). As a result of SU(3) symmetry

$$\langle \eta^0 | T j_\alpha^{eM}(x) j_\beta^{eM}(x') | 0 \rangle = \frac{2}{3} \langle \eta^0 | T j_\alpha^3(x) j_\beta^{eM}(x') | 0 \rangle . \tag{9}$$

Therefore:

$$f^2 M^\alpha(\eta^0 \to \pi^+\pi^-\gamma) = \frac{3}{2} (q - q')_\beta \, M^{\alpha\beta}(\eta^0 \to 2\gamma) . \tag{10}$$

We introduce the decay amplitudes:

$$\begin{aligned} M^\alpha(\eta^0 \to \pi^+\pi^-\gamma) &= f_{\eta^0 \to 2\pi\gamma} \, \epsilon_{\alpha\beta\gamma\delta} \, k_\beta \, q_\gamma \, q'_\delta , \\ M^{\alpha\beta}(\eta^0 \to 2\gamma) &= f_{\eta^0 \to 2\gamma} \, \epsilon_{\alpha\beta\gamma\delta} \, k_\gamma \, k'_\delta \end{aligned} \tag{11}$$

(k and k' are the γ-quantum momenta, and q and q' the pion momenta).

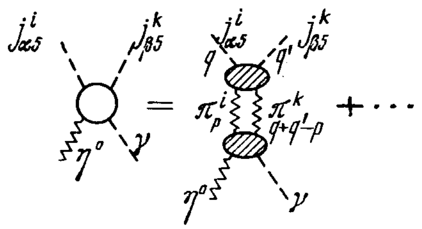

Fig. 3

We see from (11) that formula (8) is, strictly speaking, incorrect, for owing to the bilinearity of the amplitudes (11) in q, q' and k, k' the non-pole terms in the left-hand side of (6) can make a contribution comparable with the pole terms. However, the two-pion state which is the lowest in terms of mass does not yield corrections to (8), because of kinematic reasons. Indeed, let us consider the amplitude shown in Fig. 3.

The contribution from the lower block can be written in the form

$$A(p, q + q') \, \epsilon_{\gamma\nu\sigma\tau} \, (q + q')_\nu \, k_\sigma \, p_\tau$$

(p is the integration variable).

Therefore in the limit as $q, q' \to 0$ the amplitude of Fig. 3 tends to zero, and when multiplied by $q_\alpha q'_\beta$ it cannot compete with the pole term which is bilinear in q and q'. One can hope that the remaining diagrams yield only a small correction to formula (8) [3]. If we neglect this correction, then we get the following relation from (8)

$$e \, f^2 \, f_{\eta^0 \to 2\pi\gamma} = \frac{3}{2} \, f_{\eta^0 \to 2\gamma} . \tag{12}$$

From (12) we get

$$\frac{\Gamma(\eta^0 \to \pi^+\pi^-\gamma)}{\Gamma(\eta^0 \to 2\gamma)} \simeq 12\% \tag{13}$$

Experiment yields for this ratio 15%.

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1) The method used for the remainder of the proof is similar to that used in [2].

2) The amplitude f is connected with the width of the $\pi_{\mu 2}$ decay by the formula

$$\Gamma(\pi_{\mu 2}) = \frac{G^2 f^2 m_\pi}{4\pi} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2. \qquad (6)$$

The heavy lines in Fig. 2 correspond to the pions.

3) Similar reasoning can be used also to explain why the correct ratio of the decays $\omega^0 \to \pi^0 \gamma$ and $\omega^0 \to \pi^+\pi^-\pi^0$ is obtained in (3).

CURRENT COMMUTATORS AND RADIATIVE DECAYS OF THE η MESON

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On the basis of the current commutators, the following relations are obtained between the probabilities of the radiative decays of pseudoscalar mesons:

$$\frac{w(\eta \to \pi^+\pi^-\gamma)}{w(\eta \to 2\gamma)} = 0.18$$

and

$$\frac{w(X \to \pi^+\pi^-\gamma)}{w(\eta \to 2\gamma)} = 5.$$

VORTEX ISOMERS OF NUCLEI

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If nuclear matter is a superfluid liquid, then a state corresponding to a drop of this liquid is possible, i.e., a nucleus with a quantized vortex [1] passing along the axis of the drop.

The circulation of the velocity along a contour surrounding the vortex, as is well known, equals $\hbar/m$, where m is the mass of the bosons making up the superfluid liquid. This means that each such boson makes a contribution equal to $\hbar$ to the angular momentum. Consequently the total angular momentum of the nucleus in the vortical state is equal to $n\hbar = z\hbar/2$. It is assumed that the role of the bosons, whose number equals n, is played by α particles.

Since the rotation is not similar to rotation with constant angular velocity ($\omega \sim 1/r^2$ in the presence of a vortex), the equilibrium shape of the drop has the form shown in Fig. 1, with a dip on the axis. The greatest interest attaches to the minimum energy E_m of the