1) The method used for the remainder of the proof is similar to that used in [2].

2) The amplitude f is connected with the width of the $\pi_{\mu 2}$ decay by the formula

$$\Gamma(\pi_{\mu 2}) = \frac{G^2 f^2 m_\pi}{4\pi} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2. \qquad (6)$$

The heavy lines in Fig. 2 correspond to the pions.

3) Similar reasoning can be used also to explain why the correct ratio of the decays $\omega^0 \to \pi^0\gamma$ and $\omega^0 \to \pi^+\pi^-\pi^0$ is obtained in (3).

## CURRENT COMMUTATORS AND RADIATIVE DECAYS OF THE η MESON

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On the basis of the current commutators, the following relations are obtained between the probabilities of the radiative decays of pseudoscalar mesons:

$$\frac{w(\eta \to \pi^+\pi^-\gamma)}{w(\eta \to 2\gamma)} = 0.18$$

and

$$\frac{w(X \to \pi^+\pi^-\gamma)}{w(\eta \to 2\gamma)} = 5.$$

## VORTEX ISOMERS OF NUCLEI

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If nuclear matter is a superfluid liquid, then a state corresponding to a drop of this liquid is possible, i.e., a nucleus with a quantized vortex [1] passing along the axis of the drop.

The circulation of the velocity along a contour surrounding the vortex, as is well known, equals $\hbar/m$, where m is the mass of the bosons making up the superfluid liquid. This means that each such boson makes a contribution equal to $\hbar$ to the angular momentum. Consequently the total angular momentum of the nucleus in the vortical state is equal to $n\hbar = z\hbar/2$. It is assumed that the role of the bosons, whose number equals n, is played by α particles.

Since the rotation is not similar to rotation with constant angular velocity ($\omega \sim 1/r^2$ in the presence of a vortex), the equilibrium shape of the drop has the form shown in Fig. 1, with a dip on the axis. The greatest interest attaches to the minimum energy $E_m$ of the nucleus, for a given angular momentum I, as a function of I.

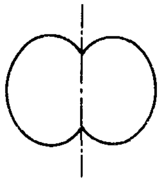

Fig. 1

A qualitative plot of $E_m(I)$ for the case of superfluidity is shown in Fig. 2. The vortical state with $I = I_0 = n\hbar = z\hbar/2$ is a sharp minimum of the $E_m(I)$ curve with a discontinuity of the derivative $\partial E_m/\partial I$, since the increment of the energy depends linearly on the modulus of the momentum, $\Delta E_m \approx k\Delta I$, for the excitations imposed on the vortical state.

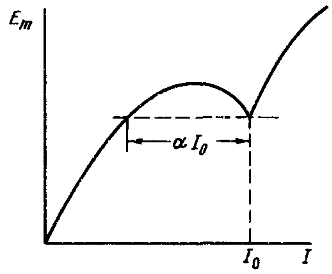


Fig. 2

It follows from the form of the curve in Fig. 2 that the vortical state can be regarded as isomeric: its energy exceeds that of the ground state, but the energy can be decreased by emission of a quantum or of some particle only by changing simultaneously the momentum by an amount equal to not less than a certain fraction $\alpha I_0$ (see Fig. 2). Therefore the transition to the ground state, direct or cascade, has low probability and the vortical state of the nucleus is expected to be long-lived.

It is of interest also to realize experimentally a vortical state of drops of liquid superfluid helium. In this case the superfluidity and the existence of quantum vortices are well known in themselves. Nonetheless, the behavior of the droplets can have curious peculiarities: it is interesting to observe the Magnus effect during falling, the change of the direction of the momentum while its magnitude is conserved, and the evaporation of a vortex drop. When it comes to the nucleus, the question may be raised regarding the smallest number of bosons above which the notions of superfluidity and quantum vortex can be employed. We note in this connection that in the single-particle treatment of the non-interacting bosons in a spherical self-consistent field the function $E_m(I)$ also has a kink, i.e., a break in the derivative, at $I = n\hbar$, $I = 2n\hbar$... [1)]. This property remains also when an interaction between bosons is turned on. This leaves only the quantitative question whether

$$\left.\frac{\partial E}{\partial I}\right|_{I = n\hbar_0 - 0} < 0,$$

in the system, as shown in Fig. 2.

Consequently, an isomer state with $I = z\hbar_0/2$ can be realized in principle also in a relatively light nucleus.

The preparation of such isomeric states is apparently most probable by collision of bulky particles, but not by the action of quanta, protons, or neutrons.


[1] R. P. Feynman, Proc. First Intern. Conf. Low Temperatures, 1958.


1) An exception is the degenerate case of the harmonic-oscillator potential $U = ar^2$.