EFFECT OF ION MOTION ALONG THE MAGNETIC FIELD ON PLASMA STABILITY

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Instabilities due to drift oscillations of an inhomogeneous plasma have been under intensive investigation during recent years [1,2]. As a rule, however, the motion of the ions along the magnetic field $\vec{H}_0$ has been little studied, and principal attention was paid to the ion currents of the magnetic field. Yet with increasing H_0 and k_z (projection of the wave vector on the magnetic-field direction) the situation can change and the longitudinal ion motion may become more important. This applies in particular, in the case of isothermal plasma, to the frequency region $\omega \lesssim k_z v_{Ti}$ (v_{Ti} = thermal velocity of the ions), or in the case of cold ions to the region $\omega \lesssim k_z v_s$ (v_s = velocity of ion sound).

We emphasize that an investigation of plasma instability at these frequencies is important in connection with the question of the effective use of installations with crossing field lines (see, e.g., the paper by Kadomtsev and Pogutse [3]). Bearing this in mind, we consider the following case: the transverse ion currents are neglected, but the longitudinal ion motion is taken into account. We shall investigate in the hydrodynamic approximation exponential perturbations (curl $\vec{E} = 0$, $\vec{E}$ = electric field of the perturbation) chosen in the form $\sim\exp(i\omega t + ik_y y + ik_z z)$.

Under the assumptions made, the equation of charge conservation is

$$v_{ze} - v_{zi} = 0, \qquad (1)$$

where v_{ze} and v_{zi} are respectively the longitudinal perturbed velocities of the electrons and ions.

To explain the main features of the phenomenon of interest to us, we consider first a simple case, when the ions are kept cold and the initial electron temperature T_0 is constant. In this case we need also the following equations:

$$i\omega M v_{zi} = sik_z T_e + eE_z, \qquad (2)$$

$$-(1 + s)ik_z T_e n_0 - ik_z n T_0 - en_0 E_z = 0, \qquad (3)$$

$$i\omega n + c(E_y/H_0)n_0' + ik_z v_{ze} n_0 = 0, \qquad (4)$$

$$(3/2)i\omega T_e + ik_z v_{ze} T_0 = -k_z^2 \chi T_e, \qquad (5)$$

$$v_x = c(E_y/H_0) + ik_y(c/eH_0)(T_e + T_0 n/n_0) \tag{6}$$

$$\left(s = 0.71;\quad n_0' \equiv \frac{dn_0}{dx}\right).$$

Here (2) - (5) are respectively the equations of ion and electron motion along $\vec{H}_0$ and the equations of continuity and heat balance for the electrons. Formula (5) takes into account expression (6) for the electron velocity along the inhomogeneity. M is the ion mass, c the velocity of light, e the electron charge, n and T_e respectively the density and electron-temperature perturbations, $n_0(x)$ the initial plasma density, and χ the coefficient of electron thermal conductivity. The terms proportional to s are connected with the thermal force - the friction force that depends on the electron temperature gradient (see the paper by Braginskii [4]) and plays a special role here. As a result we get the following dispersion equation

$$\omega^3 - \omega^2\left(\omega_e + \frac{2}{3}i\chi k_z^2\right) - \omega\left(\frac{5}{3}k_z^2 v_s^2 - \frac{2}{3}i\chi k_z^2\omega_e\right) - \frac{2}{3}s\omega_e k_z^2 v_s^2 + \frac{2}{3}ik_z^4\chi v_s^2 = 0$$

$$\left(\omega_e = k_y(cT_0/eH_0)(n_0'/n_0);\quad v_s = \sqrt{T_0/M}\right). \tag{7}$$

From (7) with $\omega_e \gg \omega$ we deduce the existence of the following instability:

$$\mathrm{Im}\,\omega \sim \sqrt{s}k_z v_s;\quad \mathrm{Re}\,\omega \sim k_z^2 v_s^2/\omega_e;\quad (\chi k_z^2 \ll k_z v_s); \tag{8}$$

$$\mathrm{Im}\,\omega \sim v_s^2/\chi;\qquad (\chi k_z^2 \gg k_z v_s). \tag{9}$$

We now turn to isothermal plasma, confining ourselves for simplicity to the case $k_z v_{Ti} \gg \chi k_z^2$. The heat-balance equation for the ions coincides with (5), for in this case the term characterizing the heat exchange between the electrons and the ions is of the order of $\chi k_z^2 T_e$ [4] and can be omitted in the approximation employed. Then, supplementing (2) with the ion pressure gradient, we can readily ascertain that the unstable root (8) remains also in this case (with v_s replaced by v_{Ti}).

As is well known [1], if $\mathrm{Im}\,\omega \sim \omega_e$ and at the same time the dimension of the turbulent pulsations is of the order of the transverse dimensions of the system, then the coefficient D of anomalous plasma diffusion due to the developing instability can become of the order of the Bohm diffusion coefficient: $D \sim cT_0/eH_0$ [5]. We call attention to the fact that if $k_z v_s \sim (1/r^2)(cT_0/eH_0)$ (r is the characteristic transverse dimension) and $\chi k_z^2 \lesssim k_z v_s$ then, as follows from (8), the instability in question will lead to Bohm diffusion.

If $\omega_e < k_z v_s$ then, as follows from (7), the instability considered here does not take place. Hence, using also (9), we can readily see that in systems with crossing force lines the coefficient becomes of the same order as the coefficient of classical diffusion if $\theta \lesssim 1$ (θ is the angle through which the force lines of H_0 are turned in a distance of the order of the dimensions of the system; $D \sim \mathrm{Im}\,\omega\Delta x^2$, where the characteristic dimension of the turbulent pulsations Δx is determined from the condition $\omega_e \sim k_{\parallel} v_s$; $k_{\parallel} \sim k_y(\theta/r)\Delta x$).

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RECONSTRUCTION OF AN IMAGE FROM A HOLOGRAM IN NONMONOCHROMATIC LIGHT

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The requirements imposed on monochromatic light for satisfactory reconstruction of an image from a hologram may be much less stringent than the conditions necessary to obtain the hologram. When a light source with relatively broad spectrum is used for the reconstruction of the image, a separate image is obtained for each wavelength. The images differ in spatial position and in scale, and this reduces the sharpness of the image and consequently leads to a loss of some of the information contained in the hologram.

It can be assumed with equal justification that the reconstruction of a hologram in nonmononchromatic light constitutes an incoherent addition of images reconstructed from individual area elements of the hologram. The dimensions of these area elements are determined by the condition that for a given spectral composition of the radiation the difference in travel between the extreme rays from such an area element must not exceed the length of the wave train at the point of the reconstructed image. The volume of information retained in the image will correspond to the information contained in one area element. The action of the entire hologram reduces in this case to an increase of the illumination and the averaging of the graininess of the image due to the limited aperture of the light beam in the case when the hologram area is small.

An elementary analysis, together with a calculation of the corresponding correlation functions, shows that the linear dimension D of the elementary hologram area, which determines the angular resolution, is given for a source of spectral width $\Delta\lambda$ by the formula

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{a}\frac{R}{D}, \tag{1}$$

where R is the distance from the point source of light to the hologram, and a is the linear dimension of the object. The same formula determines the maximum permissible spectral interval at which the information contained in a hologram of given width is completely retained in the reconstructed image.

The figure shows photographs reconstructed from a hologram obtained from a diapositive slide: (a) in laser light (λ = 6328 Å), (b) in the light of the green line obtained from a