[1] F. K. Fong, J. Chem. Phys. 41, 245 (1964).
[2] D. S. McClure and Z. Kiss, ibid. 39, 3251 (1963).
[3] F. K. Fong, Radio Corp. of Amer. Rev. 25, 303 (1964).
[4] Yu. K. Voron'ko, V. V. Osiko, V. T. Udovenchik, and M. M. Fursikov, FTT 7, 267 (1965), Soviet Phys. Solid State 7, 204 (1965).
[5] A. A. Kaminskii, L. S. Kornienko, and A. M. Prokhorov, JETP 48, 476 (1965), Soviet Phys. JETP 21, 318 (1965).

CONCERNING ONE METHOD OF ANALYZING MAGNETIC BREAKDOWN IN METALS

A. A. Slutskin
Physico-technical Institute, Ukrainian Academy of Sciences
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As is well known [1], magnetic breakdown, which is observed in many metals placed in a constant and homogeneous magnetic field $\vec{H} = \{0, 0, H\}$, is due to quantum tunneling of a charged quasiparticle (conduction electron) between closely-lying trajectories $\epsilon_s(\vec{p}) = E$, $p_z = p_{z0}$ ($\epsilon_s(\vec{p})$ = dispersion law, $\vec{p}$ = quasimomentum, s = number of band; the energy E and the projection of the momentum on the z axis are conserved), which determine the classical motion of the electron in $\vec{p}$-space. In constructing a quantitative theory of magnetic breakdown, greatest interest is attached to the calculation of the probability of interband breakdown and the determination of the jump in phase of the quasiclassical wave function, a jump occurring when the electron goes from one trajectory to another. In the quasiclassical approximation, which is well satisfied in metals, these two quantities determine completely the energy spectrum and the wave function of the quasiparticle and enter in all the macroscopic electronic characteristics of the metal.

In this note we propose to analyze interband magnetic breakdown by a method which yields simple analytic expressions for the breakdown probability and the phase difference; these equations are valid in the entire interval of magnetic-field variation [1)]. In accord with the presently held view concerning charged quasiparticles as the carriers of conductivity in metals, the analysis is carried out in terms of an arbitrary dispersion law.

We shall carry out our investigation for a case when the points in $\vec{p}$-space, where the difference $\epsilon_1(\vec{p}) - \epsilon_2(\vec{p}) = \Delta(\vec{p})$ is small, are concentrated near a certain plane. (This situation is typical of metals.) For simplicity we can assume that at fixed p_x and p_z the minimum of $\Delta(\vec{p})$ ($\min \Delta(\vec{p}) \neq 0$) is reached on the plane $p_y = 0$.

In regions of $\vec{p}$-space that are sufficiently remote from the plane $p_y = 0$, we can neglect interband transitions and the wave function $G_s(\vec{P})$ of the electron in the $\vec{p}$,s representation (defined as the coefficients of the expansion in terms of the functions $\tilde{\psi}_{\vec{p},s} = u^{(s)}_{p_x+eH_y/c,p_y,p_z}(\vec{r})\exp(i\vec{p}\cdot\vec{r}/\hbar)$, where $u^{(s)}_{\vec{p}}(\vec{r})$ is the periodic multiplier in the Bloch function), written in the quasiclassical approximation, and is of the form

$$G_s(\vec{P}) = g_s(P_y)\delta(P_x - P_{x0})\delta(P_z - p_{z0}); \qquad (1)$$

$$g_s^{(\pm)}(P_y) = \sum_\alpha c_{s,\alpha}^{(\pm)} \exp \frac{i}{\sigma} \left(P_{x0}P_y - \int_0^{P_y} p_x^{(\alpha,s)}(p_y')dp_y'\right) / \sqrt{|\partial\epsilon_s/\partial p_x|},$$

$$\epsilon_s(p_x^{(\alpha,s)}(p_y),\ p_y,\ p_{z0}) = E, \quad \sigma = e\hbar H/c. \qquad (1a)$$

The plus and minus signs pertain to the regions of $\vec{p}$-space where $p_y > 0$ and $p_y < 0$, respectively, and the index α denotes one of the solutions of Eq. (1a). The problem of magnetic breakdown reduces in fact to a determination of the "joining" matrices $\hat{\tau}^{(\alpha)}$ that relate the coefficients $c_{s,\alpha}^{(\pm)}$ $(s = 1,2)$:

$$c_{s,\alpha}^{(+)} = \sum_{s'=1}^{2} \tau_{ss'}^{(\alpha)} c_{s',\alpha}^{(-)} .$$

To obtain $\hat{\tau}^{(\alpha)}$ in explicit form as a function of H and of the main parameters of the problem, it is necessary to investigate the Schrodinger equation in a small vicinity of $p_y = 0$ (where interband transitions are significant) and "join" the obtained solution to the quasiclassical expressions (1). The functions $\tilde{\psi}_{\vec{p},s}$ are not suitable for this investigation, since the periodic factors $u_{\vec{p}}^{(s)}(\vec{r})$ contained in these functions are "sharp" with respect to the parameter p_y (the characteristic interval of variation of $u_{\vec{p}}^{(s)}$ is of the order of $(\Delta_0/\epsilon_0)p_0 \ll p_0 = \hbar/a$, where a is the lattice constant, ϵ_0 the characteristic energy, and Δ_0 the characteristic value of $\Delta(p_x,\ 0,\ p_z)$). It is more expedient to use in lieu of $\tilde{\psi}_{\vec{p},s}$ the functions $\chi_{\vec{p},s} = \tilde{\psi}_{p_x,0,p_z}\exp(ip_y y/\hbar)$, where the dependence on p_y enters only in the exponential function (χ are modified Kohn-Luttinger functions). Simple calculations carried out in the first approximation in the quasiclassical parameter show that for small p_y the Schrodinger equation in the χ representation is

$$\sum_{s'=1}^{2} \{[\hat{\epsilon}_s(\hat{p}_x,\ 0,\ \hat{p}_z) - E]\delta_{ss'} + p_y\hat{v}_{ss'}(\hat{p}_x,\ 0,\ \hat{p}_z)\}\beta_s(\vec{P}) = 0, \quad \hat{p}_x = P_x + i\sigma\frac{\partial}{\partial P_y}. \qquad (2)$$

Here $\beta_s(\vec{P})$ are the coefficients of the expansion of the complete electron wave function in powers of $\chi_{\vec{p},s}$, the connection between β_s and G_s on the right and on the left of the $p_y = 0$ plane is determined by matrices that are independent of p_y, and $v_{ss'}(\vec{p})$ is a matrix element of the velocity operator $\hat{v}_y$. Solution of the system in the first approximation in the parameter [2)] $\hbar\Omega_0/\Delta_0 \ll 1$ (Ω_0 is the characteristic value of the Larmor frequency) makes the "joining" possible and yields an explicit form of the matrix $\hat{\tau}$:

$$\hat{\tau}^{(\alpha)} = \begin{pmatrix} \tau_\alpha \exp(i\omega_\alpha), & -s_\alpha \\ s_\alpha, & \tau_\alpha\exp(-i\omega_\alpha) \end{pmatrix}, \quad \begin{array}{l} \tau_\alpha^2 + s_\alpha^2 = 1,\ s_\alpha = \exp(-\pi\gamma_\alpha/2),\ \gamma_\alpha = \Delta_{0\alpha}^2/\sigma|v_{x0}^{(\alpha)}V_0^{(\alpha)}|, \\ \omega_\alpha = \text{sign}\, v_{x0}^{(\alpha)}\arg\left\{\exp i\left(\frac{\pi}{4} + \frac{\gamma_\alpha}{2} - \frac{\gamma_\alpha}{2}\ln\frac{\gamma_\alpha}{2}\right)/\Gamma^*\left(\frac{i\gamma_\alpha}{2}\right)\right\}, \end{array} \qquad (3)$$

$$V = v_{12},\ v_x = \tfrac{1}{2}\sum_{s=1}^{2}\partial\epsilon_s/\partial p_x.$$

The zero subscript means that all the quantities in (3) are taken on the $p_y = 0$ plane at the maximum-breakdown point $(p_{x0}, 0, p_{z0})$, which is determined by the equation

$$\frac{1}{2}\sum_{s=1}^{2}\epsilon_s(p_{x0}, 0, p_{z0}) = E.$$

The matrices $\hat{\tau}^{(\alpha)}$ are unitary, corresponding to conservation of the quantity

$$\sum_{s=1}^{2}(\partial\epsilon_s/\partial p_x)|g_s|^2,$$

which is proportional to the probability flux density in $\vec{p}$-space. ω_α determines the jump in phase of the quasiclassical wave function. The magnetic-breakdown probability W_α is by definition equal to $|\tau_{12}^{(\alpha)}| = |\tau_{21}^{(\alpha)}|$, that is to say,

$$W_\alpha = \exp(-H_0(E, p_{z0})/H), \qquad H_0(E, p_{z0}) = \frac{c\Delta_{0\alpha}^2}{eh|v_{x0}^{(\alpha)}V_0^{(\alpha)}|}. \tag{4}$$

Formula (4) is valid for arbitrary H. If the magnetic field is inclined by a certain angle φ to the plane of closest approach of the bands, then W_α is given by

$$W_\alpha = \exp(-H_0(E, p_{z0})/H\cos\varphi), \tag{4a}$$

i.e., W_α decreases with increasing φ.

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[1] H. M. Cohen and L. M. Falicov, Phys. Rev. Lett. 7, 231 (1961).
[2] E. J. Blount, Phys. Rev. 126, 1636 (1962).
[3] A. B. Pippard, Proc. Roy. Soc. 270, 1 (1962); Phil. Trans. Roy. Soc. (London) 256, 317 (1964).
[4] I. M. Lifshitz and M. I. Kaganov, UFN 69, 419 (1959), Soviet Phys. Uspekhi 2, 831 (1960).

1) This problem was solved by Blount [2] and by Pippard [3] for the limiting cases of very strong ($H \to \infty$) and weak ($H \to 0$) breakdown.

2) The smallness of $\hbar\Omega_0/\Delta_0$ does not contradict the condition of even very strong breakdown, and imposes no limitations on our analysis.

INFRARED AND MICROWAVE ABSORPTION IN IONIC CRYSTALS AT HIGH POWER LEVELS

V. M. Fain
Radiophysics Research Institute in Gor'kii
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The purpose of this communication is to demonstrate that anomalously large absorption