

The zero subscript means that all the quantities in (3) are taken on the  $p_y = 0$  plane at the maximum-breakdown point  $(p_{x0}, 0, p_{z0})$ , which is determined by the equation

$$\frac{1}{2} \sum_{s=1}^2 \epsilon_s(p_{x0}, 0, p_{z0}) = E.$$

The matrices  $\hat{\tau}^{(\alpha)}$  are unitary, corresponding to conservation of the quantity

$$\sum_{s=1}^2 (\partial \epsilon_s / \partial p_x) |g_s|^2,$$

which is proportional to the probability flux density in  $\vec{p}$ -space.  $\omega_\alpha$  determines the jump in phase of the quasiclassical wave function. The magnetic-breakdown probability  $W_\alpha$  is by definition equal to  $|\tau_{12}^{(\alpha)}| = |\tau_{21}^{(\alpha)}|$ , that is to say,

$$W_\alpha = \exp(-H_0(E, p_{z0})/H), \quad H_0(E, p_{z0}) = \frac{c\Delta_{0\alpha}^2}{eh|v_{x0}^{(\alpha)}v_0^{(\alpha)}|}. \quad (4)$$

Formula (4) is valid for arbitrary  $H$ . If the magnetic field is inclined by a certain angle  $\varphi$  to the plane of closest approach of the bands, then  $W_\alpha$  is given by

$$W_\alpha = \exp(-H_0(E, p_{z0})/H \cos\varphi), \quad (4a)$$

i.e.,  $W_\alpha$  decreases with increasing  $\varphi$ .

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1) This problem was solved by Blount [2] and by Pippard [3] for the limiting cases of very strong ( $H \rightarrow \infty$ ) and weak ( $H \rightarrow 0$ ) breakdown.

2) The smallness of  $\hbar\Omega_0/\Delta_0$  does not contradict the condition of even very strong breakdown, and imposes no limitations on our analysis.

#### INFRARED AND MICROWAVE ABSORPTION IN IONIC CRYSTALS AT HIGH POWER LEVELS

V. M. Fain  
 Radiophysics Research Institute in Gor'kii  
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The purpose of this communication is to demonstrate that anomalously large absorption

(in the nonresonant case) and saturation of infrared resonance take place when an external electric field of sufficiently large amplitude is applied to a dielectric. These phenomena are connected with the occurrence of parametric instability of acoustic waves [1] and are analogous to the corresponding phenomena in ferromagnetic resonance at high power levels [2]. The equations for the stationary values of the amplitudes of the acoustic waves,  $a_{\mathbf{k}}$  and  $a_{-\mathbf{k}}^+ = a_{\mathbf{k}}^*$ , and of the optical oscillations,  $a$  and  $a^+ = a^*$  (at  $\vec{k} = 0$ ), are

$$[i(\omega_{\mathbf{k}} - \omega/2) + \gamma_{\mathbf{k}}]a_{\mathbf{k}} = -iF_{\mathbf{k}}x a_{-\mathbf{k}}^+, \quad (1)$$

$$[-i(\omega + \omega_0) + \gamma]a_- = i\sum_{\mathbf{k}} F_{\mathbf{k}} a_{\mathbf{k}} a_{-\mathbf{k}} - i\beta E_0, \quad (2)$$

$$[-i(\omega - \omega_0) + \gamma]a_-^+ = -i\sum_{\mathbf{k}} F_{\mathbf{k}} a_{\mathbf{k}} a_{-\mathbf{k}} + i\beta E_0.$$

Here  $x = a_-^+ + a_-$ , where  $a_-^+$  and  $a_-$  are the amplitudes at  $\exp(i\omega t)$ . Equations (1) and (2) follow from the Hamiltonian that describes the phonons in an external homogeneous field  $E_x = E_0 \cos \omega t$ , with allowance for third-order anharmonicity, and ensures a connection  $V_{0\mathbf{k},-\mathbf{k}} \equiv \mathfrak{M}F_{\mathbf{k}}$  between the acoustic and optical phonons.

A stable solution of Eqs. (1) and (2) in the classical case, neglecting thermal fluctuations, is

$$x = -\frac{2\beta\omega_0 E_0}{\omega^2 - \omega_0^2 - \gamma^2 + i\omega\gamma}; \quad a_{\mathbf{k}} = 0 \text{ if } E_0 < E_{\text{cr}},$$

$$x = -\frac{2\beta\omega_0 E_0}{\omega^2 - \omega_0^2 - \gamma^2 + \omega_0 R_2 + i\omega(\gamma + 2(\omega_0/\omega)R_1)}; \quad a_{\mathbf{k}} \neq 0, \quad (3)$$

$$|x^2| = \min \frac{(\omega_{\mathbf{k}} - \omega/2)^2 + \gamma_{\mathbf{k}}}{|F_{\mathbf{k}}|^2} \quad \text{if } E_0 \geq E_{\text{cr}},$$

where the minimum is taken of the expression in the right side, regarded as a function of  $\vec{k}$ ,

$$R_1 = 2 \sum_{\mathbf{k}} \frac{\gamma_{\mathbf{k}}}{(\omega_{\mathbf{k}} - \omega/2)^2 + \gamma_{\mathbf{k}}^2} |F_{\mathbf{k}}|^2 n_{\mathbf{k}}; \quad R_2 \equiv \alpha R_1 = 2 \sum_{\mathbf{k}} \frac{(\omega/2) - \omega_{\mathbf{k}}}{(\omega_{\mathbf{k}} - \omega/2)^2 + \gamma_{\mathbf{k}}^2} |F_{\mathbf{k}}|^2 n_{\mathbf{k}},$$

$n_{\mathbf{k}} = a_{\mathbf{k}}^+ a_{\mathbf{k}}$ , and the summation is over all  $\vec{k}$  satisfying the condition for the minimum. Instability of the acoustic waves sets in when

$$E_0^2 > E_{\text{cr}}^2 = \frac{(\omega^2 - \omega_0^2 - \gamma^2)^2 + \omega^2 \gamma^2}{4\beta^2 \omega_0^2} \min \frac{(\omega_{\mathbf{k}} - \omega/2)^2 + \gamma_{\mathbf{k}}^2}{|F_{\mathbf{k}}|^2}.$$

It is easy to obtain the following estimates:

$$\beta \sim e\sqrt{N/M\omega_0}; \quad F_{\mathbf{k}} \sim \sqrt{\mathfrak{M}\omega_0/NM} (\omega_0/a\omega_{\mathbf{k}})(a\mathbf{k})^2$$

and when  $\omega \ll \omega_0$  (in the microwave band)  $E_{\text{cr}} \sim M\omega_{\mathbf{k}}\gamma_{\mathbf{k}}/eak^2$ , where  $M$  is the average ion mass and  $a$  the lattice constant.  $E_{\text{cr}} \sim 10^{-7}\gamma_{\mathbf{k}}$  when  $\omega_{\mathbf{k}} \sim \omega/2 \sim 10^{11} \text{ sec}^{-1}$ ,  $k \sim 10^8 \text{ cm}^{-1}$  and  $M \sim 10^{-23} \text{ g}$ , so that  $E_{\text{cr}} \sim 1 \text{ cgs esu} = 300 \text{ V/cm}$  when  $\gamma_{\mathbf{k}} \sim 10^7 \text{ sec}^{-1}$ . Such field intensities

are perfectly feasible in the microwave band. We now determine the dielectric susceptibility above the threshold  $E_0 > E_{cr}$ . According to (3)  $|x|^2$  does not depend in this case on  $E_0$

$$|x|^2 = \frac{4\beta^2 \omega_0^2 E_0^2}{(\omega^2 - \omega_0^2 - \gamma^2 + \omega_0 R_2)^2 + \omega^2 (\gamma + 2(\omega_0/\omega) R_1)^2} = \frac{4\beta^2 \omega_0^2 E_{cr}^2}{(\omega^2 - \omega_0^2 - \gamma^2)^2 + \omega^2 \gamma^2}. \quad (4)$$

Using this equation, it is easy to express  $R_1$  and  $R_2$  in terms of the power ratio of the incident radiation power and the critical power,  $W/W_{cr} \geq 1$ , and in terms of the parameter  $\alpha$

$$R_1 = - \frac{2\alpha\omega_0\gamma - \alpha\omega_0(\omega^2 - \omega_0^2 - \gamma^2)}{(4 + \alpha^2)\omega_0^2} + \left[ \frac{[2\alpha\omega_0 - \alpha\omega_0(\omega^2 - \omega_0^2 - \gamma^2)]^2}{(4 + \alpha^2)\omega_0^4} + \left(\frac{W}{W_{cr}} - 1\right) \frac{(\omega^2 - \omega_0^2 - \gamma^2)^2 + \omega^2 \gamma^2}{(4 + \alpha^2)\omega_0^2} \right]^{\frac{1}{2}}; \quad R_2 = \alpha R_1. \quad (5)$$

The parameter  $\alpha$  is determined by the ratio  $(\omega_k - \omega/2)/\gamma_k$  and, generally speaking, is small. It is of the order of  $\gamma_k/\omega_k$  for an isotropic-crystal model and can be neglected if we exclude the small region  $W/W_{cr} - 1 < \alpha^2/4$ . In this case and for  $W > W_{cr}$  the susceptibilities can be expressed in terms of the subthreshold susceptibility as follows:

$$\chi'_{W>W_{cr}} = \chi'_{W<W_{cr}} \frac{W_{cr}}{W}; \quad \chi''_{W>W_{cr}} = \chi''_{W<W_{cr}} \frac{W_{cr}}{W} \left[ \frac{W}{W_{cr}} + \left(\frac{W}{W_{cr}} - 1\right) \frac{(\omega^2 - \omega_0^2 - \gamma^2)^2}{\omega^2 \gamma^2} \right]^{\frac{1}{2}} \quad (6)$$

(see the analogous formulas in [2]).

As is well known from the theory of ferromagnetic resonance [2], the approximation used above, which does not take fluctuations into account, is quite satisfactory at high power levels. Near the instability threshold, however, allowance for the fluctuations is essential. It is therefore meaningful to present results from the theory with allowance for fluctuations. We must start then from the system of quantum equations for the rms quantities  $n_k = \langle a_k^+ a_k \rangle$  and  $\langle a_k a_{-k} \rangle = A_k \exp(-i\omega t)$ . In the stationary state, with allowance for only the resonant terms, these equations take the form <sup>1)</sup>

$$\begin{aligned} 2\gamma_k(n_k - n_k^0) &= -iF_k x A_k^* + iF_k^* x^* A_k, \\ [i(\omega - 2\omega_k) + 2\gamma_k] A_k &= -iF_k x (2n_k + 1), \end{aligned} \quad (7)$$

where  $n_k^0$  is the mean thermodynamic value of  $n_k$ . The solution of these equations is

$$\begin{aligned} (2n_k + 1) &= (2n_k^0 + 1) \frac{(\omega - 2\omega_k) + 4\gamma_k}{(\omega - 2\omega_k)^2 + 4\gamma_k^2 - 4|F_k|^2|x|^2}, \\ A_k &= - \frac{F_k x (2n_k^0 + 1) [(\omega - 2\omega_k) + 2i\gamma_k]}{(\omega - 2\omega_k)^2 + 4\gamma_k^2 - 4|F_k|^2|x|^2}. \end{aligned} \quad (8)$$

From (2) we can now obtain a relation between  $x$  and the external field  $E_0$

$$\left\{ \omega^2 - \omega_0^2 - \gamma^2 - i\omega\gamma + 2\omega_0 \sum_k \frac{|F_k|^2 (2n_k^0 + 1) (\omega - 2\omega_k + 2i\gamma_k)}{(\omega - 2\omega_k)^2 + 4\gamma_k^2 - 4|F_k|^2 |x|^2} \right\} x = -2\beta\omega_0 E_0. \quad (9)$$

The last equation replaces the solution of (3) and, as seen from a simple analysis, the solution of formulas (3) and (6), being much simpler than (9), is at the same time a good approximation of the latter. Therefore the rms fluctuations described by (8) can be obtained by substituting  $x$  from (3) (below the instability threshold and excluding a small region near threshold).

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1) Allowance for the dissipation terms is by means of the procedure described in the book of Fain and Khanin [3], p. 86. The amplitudes  $a$  and  $a^+$  of the optical oscillation are treated classically.

#### INVESTIGATION OF THE MAGNETIC FIELD OF A SPARK PRODUCED BY FOCUSING LASER RADIATION

V. V. Korobkin and R. V. Serov  
 P. N. Lebedev Physics Institute, USSR Academy of Sciences  
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The spark produced when a sufficiently powerful laser beam is focused was investigated in a number of recent papers [1-4].

Raizer [5] investigated the crowding out of a longitudinal magnetic field by the plasma of the spark, as a result of the diamagnetism of the plasma.

We have observed the spark's own magnetic field, which exists only during the time when the plasma is fed by the laser beam. A Q-switched ruby laser was used in the experiment. The pulse power was 2 J and the pulse duration 30 nsec.

The magnetic field of the spark was measured with coils of 10 mm diameter, each consisting of two turns of wire. The signals from the two coils, which were disposed in various manners relative to the spark, passed through two different delay lines (cables 20 and 50 m long), amplified by two UZ-4 amplifiers, and displayed on an SI-11 oscilloscope. The delay-time difference was 150 nsec, so that it was possible to measure simultaneously arbitrarily chosen components of the magnetic fields at two points of space on a single oscillogram.

To suppress the photoeffect from the inductive pickups, the spark was surrounded by a tube of black paper of 5 mm inside diameter. In addition, the signal from each pickup was fed to the input of the delay line through a special isolating transformer with a grounded