was to call attention to NMR as an effective tool for the study of the structure and distribution of the internal fields in superconductors of the second kind.

In conclusion I am grateful to M. I. Kaganov for discussions.

[1] I. O. Kulik, JETP Letters 3, 183 (1966), transl. p. 116.
[2] I. O. Kulik, JETP 50, 1617 (1966), Soviet Phys. JETP 23, in press.
[3] A. A. Abrikosov, JETP 32, 1442 (1957), Soviet Phys. JETP 5, 1174 (1957).
[4] A. Loesche, Nuclear Induction, Chapter 2, IIL, 1963.

DISTRIBUTION FUNCTION OF DISTANCES BETWEEN ENERGY LEVELS OF AN ELECTRON IN A ONE-DIMENSIONAL RANDOM CHAIN

V. L. Pokrovskii
Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 4 June 1966
ZhETF Pis'ma 4, No. 4, 140-144, 15 August 1966

The distribution of level spacing in systems that are in some sense random has recently again attracted persistent interest on the part of the theoreticians. Physical examples of such systems are atomic nuclei in strongly excited states [1,2], and small metallic particles [3,4].

Random systems are presently described phenomenologically. Dyson [2] has proposed that level spacing can be described by one of three possible ensembles (unitary, symplectic, orthogonal), depending on the symmetry properties of the system. A similar initial hypothesis is used by Gor'kov and Eliashberg [4]. It is assumed in this case that these ensembles correspond to maximally randomized systems. It is very attractive to attempt to find, starting from the general principles of dynamics and probability theory, arguments in favor of Dyson's distributions, of at least the same type as already exist for the Gibbs distribution. In addition, it would be interesting to ascertain which ensembles describe level distribution for incompletely random systems.

In this paper we investigate the simplest one-dimensional model for which it is possible to obtain an explicit solution of the problem of the distribution of distances between energy levels. The obtained distribution has no similarity whatever to the Dyson distribution [2]. The character of the distribution (very narrow Gaussian peaks) is apparently connected in its essential features with the assumed simplifications of the model (one-dimensionality, absence of interactions between "electrons"). Since, however, this is the only known example where the problem is solved exactly, its results are also of interest in themselves.

Let us consider a one-dimensional chain of potential centers between which a quantum particle (electron) moves. For simplicity let us assume that the effective radius of the center is much smaller than the average distance between centers. In the zeroth approximation we can assume that the electron moves in a field with a potential:

$$V(x) = a\sum_{n=1}^{N} \delta(x - x_n). \tag{1}$$

The distances between the peaks $\ell_n = x_{n+1} - x_n$ differ by random quantities which obey a specified distribution law $P(\ell)$. It is assumed further that the correlation functions $P(\ell_n, \ell_{n'}) - P(\ell_n)P(\ell_{n'})$ decrease sufficiently rapidly with the "distance" $|n - n'|$. The meaning of this condition will be defined more accurately later.

The exact formulation of the problem is as follows: Let there be given a configuration Γ - arrangement of the points $x_1, x_2, \ldots x_N$. For the given configuration Γ, the Schrodinger equation

$$\frac{d^2\psi}{dx^2} + (k^2 - V(x))\psi = 0 \tag{2}$$

with $V(x)$ specified by formula (1) and with the boundary conditions

$$\psi(x_1) = \psi(x_N) = 0 \tag{3}$$

has a set of discrete eigenvalues k_n. The problem consists in calculating the distribution function $Q(k, k')$, averaged over all Γ, of the distances between the neighboring eigenvalues. In other words, it is required to find the probability $Q(k, k')dk'$ that if k^2 is an energy level, then the level having the next higher number lies between k'^2 and $k'^2 + 2k'dk'$.

So far, only the question of the number of eigenvalues in a specified interval $(k, k + dk)$, averaged over all the configurations Γ, has been considered in the past [5-11].

Since the potential $V(x)$ and the boundary conditions (2) are real, the wave function ψ is also real. In the interval (x_n, x_{n+1}) it can be written in the form

$$\psi = A_n \sin[k(x - x_n) + \varphi_n]. \tag{4}$$

It is easy to find the connection between φ_n and φ_{n+1}:

$$\cot\varphi_{n+1} = \cot(\varphi_n + k\ell_n) + \epsilon; \quad \epsilon = a/k. \tag{5}$$

Putting $x_1 \equiv 0$ and $\varphi_1 = 0$, we satisfy one of the boundary conditions (2). The second condition is satisfied if we put

$$\varphi_{N+1} = m\pi, \tag{6}$$

where m is an integer. If we agree that $\varphi_n + k\ell_n$ differs from φ_{n+1} by not more than $\pm\pi$, then the number m in (6) coincides with the number of zeroes of the wave function, i.e., with the number of eigenvalues smaller than a specified k, or with the number of the level.

Thus, let the condition (6) be satisfied. Our problem reduces to the following: find the probability that

$$\varphi_{N+1}(k') < (m + 1)\pi) < \varphi_{N+1}(k' + dk'). \tag{7}$$

We use the readily-proved monotonicity of φ_n as a function of k for specified Γ. We note that φ_{N+1} for arbitrary k and Γ can be represented in the form of the sum

$$\varphi_{N+1}(k, \Gamma) = \sum_{n=1}^{N} \Delta_n(k, \Gamma); \quad \Delta_n(k, \Gamma) = \varphi_{n+1}(k, \Gamma) - \varphi_n(k, \Gamma). \tag{8}$$

We represent the difference similarly:

$$\varphi_{N+1}(k', \Gamma) - \varphi_{N+1}(k, \Gamma) = (k' - k)\sum_{n=1}^{N} \partial\Delta_n(k, \Gamma)/\partial k. \tag{9}$$

We can confine ourselves to terms linear in $(k' - k)$ only when

$$\frac{N\bar{\Delta}(k' - k)^2}{k^2} \ll 1.$$

But $N\bar{\Delta}/k$ coincides in order of magnitude with $\nu(k) = \partial\bar{m}/\partial k$, so that our estimate denotes simultaneously the following:

$$\frac{(k' - k)^2}{k} \ll [\nu(k)]^{-1}. \tag{10}$$

The estimate (10) can be satisfied even in the case when many levels are contained between k and k': $(k' - k) \gg [\nu(k)]^{-1}$.

The condition of (6) and (7) can now be formulated as follows: find the probability that

$$\frac{\pi}{k' + dk' - k} < \sum_{n=1}^{N} \frac{\partial\Delta_n(k, \Gamma)}{\partial k} < \frac{\pi}{k' - k} \tag{11}$$

under the condition that $\varphi_N(k, \Gamma) = m\pi$, where m is an arbitrary integer. It has been established in [6] that for the phase φ_n, reckoned on a circle from 0 to π, a stationary distribution is established at large values of n. The correlation between φ_n and $\varphi_{n'}$ (which are close in number) decreases rapidly with increasing "distance" $|n - n'|$. Therefore the sum in (11) obeys a Gaussian distribution of the type

$$W(\Sigma)d\Sigma = A \exp\left\{- \frac{\left(\Sigma - N\frac{\partial\bar{\Delta}}{\partial k}\right)^2}{N\lambda^2}\right\}; \quad \lambda^2 = \sum_{n=-\infty}^{\infty} \overline{\frac{\partial\Delta_0}{\partial k}\frac{\partial\Delta_n}{\partial k}}. \tag{12}$$

We imply here averages over the stationary distributions as indicated above, and A is a normalization constant. According to (11) and (12), the sought probability is

$$Q(k, k')dk' = A \exp\left\{-\left(\frac{k' - k}{\overline{\Delta k}} - 1\right)^2 / N(k' - k)^2\lambda^2\right\} \frac{\pi}{(k' - k)^2} dk', \tag{13}$$

where $\overline{\Delta k} = [\nu(k)]^{-1}$ is the average distance between neighboring eigenvalues. Thus, the distribution has a Gaussian character with a narrow peak (of width $\sim N^{-\frac{1}{2}}$) about the mean value.

[1] E. P. Wigner, Ann. Math. 53, 36 (1951); 62, 548 (1955); 65, 203 (1957); 67, 325 (1958).

[2] F. J. Dyson, J. Math. Phys. 3, 140, 157, 166 (1962).

[3] R. Kubo, J. Phys. Soc. Japan 17, 975 (1962).

[4] L. P. Gor'kov and G. M. Eliashberg, JETP 48, 1407 (1965), Soviet Phys. JETP 21, 940

(1965).

[5] F. J. Dyson, Phys. Rev. 92, 133 (1953).

[6] H. Schmidt, ibid. 105, 425 (1957).

[7] H. L. Frish and S. P. Lloyd, ibid. 120, 1175 (1960).

[8] E. W. Montroll and R. B. Potts, ibid. 100, 525 (1955).

[9] I. M. Lifshitz, UFN 83, 617 (1964), Soviet Phys. Uspekhi 7, 549 (1965).

[10] G. M. Zaslavskii and V. L. Pokrovskii, JETP 51, 449 (1966), Soviet Phys. JETP 24, in press.

[11] Yu. A. Bychkov and A. M. Dykhne, JETP Letters 3, 313 (1966), transl. p. 202.

SELF-FOCUSING AND FOCUSING OF ULTRASOUND AND HYPERSOUND

G. A. Askar'yan
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 8 June 1966
ZhETF Pis'ma 4, No. 4, 144-147, 15 August 1966

Recently new sources of intense ultrasonic and hypersonic waves have become available; these are powerful laser beams, which produce in a medium a flux of volume waves that cause induced Mandel'shtam-Brillouin scattering.

We point out in this article the possibility of self-focusing and focusing of hypersonic rays from these or other sources, and estimate the conditions for the appearance and possible consequences of these effects.

The effects under consideration are based on nonlinear processes that produce a differential in the properties of the medium inside and outside the sound ray. In particular, the effects of self-focusing of sound recall the nonlinear effects of self-focusing on electromagnetic rays in media [1-6].

1. Focusing of Sound Ray by the "Wake" of a Light Ray

A light ray may modify the properties of the medium enough to change the propagation o: a sound wave. For example, absorption of light and heating of the medium change the velocit; and the propagation of sound in those portions of the medium through which the light has passed (the so-called "wake" of the light ray). In dense media (liquids, solids) the speed of sound usually decreases when energy is released in the medium: $dc_s/dT < 0$, so that the thermal wake of the ray or of part of the ray with increased intensity (light filament) can serve as a sound conductor, reflecting sound on the boundary of the wake.

If the glancing angle φ between the direction of incidence and the layer of discontinuity on the boundary of the heated region is such that $\cos\varphi > c_{s.inside}/c_{s.outside}$, i.e., for small values of $\varphi < \sqrt{-\Delta c_s/c_s}$, then total internal reflection of the sound will occur and the light wake will serve as an acoustic waveguide. Usually $\Delta c_s/\Delta T \sim -k \times 10^2$ cm sec^{-1}de£ where k is of the order of several times unity. For example, for $\Delta T \sim 0.1°$ and $c_s \simeq 10^5$ cm/sec we obtain