In the region of the ordering temperature, a change takes place also in the character of the $\lambda(H)$ dependence. In Fig. 3 we show by way of an example the isotherms of the magnetostriction of $Dy_3Ga_5O_{12}$, which show the quadratic growth of magnetostriction with magnetic field characteristic of ferromagnets. Deviation from this relation takes place already at $T = 4.2°K$, and at 2.5°K the inclination of the curve relative to the field axis reverses. A similar situation is observed in the behavior of even magnetic effects when the Curie point of a ferromagnet is approached from the high-temperature side [5].

One cannot exclude, however, the possibility that the character of the magnetostriction isotherms at low temperatures, which we have described above, is connected with paramagnetic saturation in the strong magnetic field.

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INFLUENCE OF THERMAL EXPANSION ON THE SINGULARITIES OF THE KINETIC COEFFICIENTS AT THE CURIE POINT

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1. Near second-order phase-transition points and critical points, the fluctuations increase appreciably. Variation of the character of the scattering of the carriers gives rise to singularities of the kinetic coefficients; these were calculated in many papers within the framework of the Landau theory [1-4].

Besides such a singularity, which is essentially connected with the mechanism of the phase transition and with the spectral structure responsible for the transition, the kinetic coefficients should also exhibit singularities that do not depend on the details of the scattering process and which are the consequence of singularities of the thermodynamic mean values. We consider below a singularity resulting from the dependence of the carrier dispersion law on the lattice constant, and through it - owing to the thermal expansion of the crystal - on the temperature. For concreteness we shall speak of electrons and electric conductivity. If we assume the non-deformed state of the crystal to occur at zero temperature (this choice, of course, is perfectly arbitrary), then in the approximation linear in u_{ik} the dispersion law $\epsilon(\vec{p}, T)$ can be represented in the form of an expansion about the dispersion law $\epsilon_0(\vec{p})$ at $T = 0$:

$$\epsilon = \epsilon_0(\vec{p}) + \Lambda_{ik}(\vec{p})u_{ik}(T).$$

Here $u_{ik}(T)$ is the temperature deformation of the lattice and $\Lambda_{ik}(\vec{p})$ is the renormalized deformation-potential tensor. The renormalization is connected with the inclusion of the change in the chemical potential, due to the deformation in the dispersion law; $\vec{p}$ is the quasimomentum of the electron. Solving the kinetic equation for the electron distribution function in the approximation linear in u_{ik}, we find the change in the conductivity $\Delta\sigma_{ik}$ connected with the temperature deformation:

$$\Delta\sigma_{ik} = \Sigma_{ik\ell m}\, u_{\ell m}\,(T), \qquad \sigma_{ik} = \sigma^0_{ik} + \Delta\sigma_{ik}, \qquad \Sigma \sim \sigma_0. \tag{1}$$

Only the explicit form of σ^0_{ik} and $\Sigma_{ik\ell m}$ depends on the model; in the relaxation-time (τ) approximation, for example, these are equal to

$$\sigma^0_{ik} = e^2\langle \tau v_i v_k\rangle, \quad \Sigma_{ik\ell m} = e^2\langle \frac{\delta\tau}{\delta u_{\ell m}} v_i v_k\rangle + e^2\langle \tau\left(\frac{\partial\Lambda_{\ell m}}{\partial p_i} v_k - v_i \frac{\partial\Lambda_{\ell m}}{\partial p_k} \right)\rangle.$$

The term $\delta\tau/\delta u_{\ell m}$ takes into account the influence of the deformation on the collision integral (for example, due to the change in the phonon dispersion law during collisions between electrons and phonons, etc.). Noting that $du_{ik}/dT = \alpha_{ik}$ is the coefficient of thermal expansion of the crystal, we obtain

$$\frac{d\sigma_{ik}}{dT} = \frac{d\sigma_0}{dT} + \frac{d\Sigma_{ik\ell m}}{dT} u_{\ell m}(T) + \Sigma_{ik\ell m}\alpha_{\ell m}(T). \tag{2}$$

The last term has a singularity at the transition point, since α_{ik} behaves near the Curie point, as is well known, in the same manner as the specific heat, i.e., it experiences a discontinuity and possibly has a logarithmic singularity [5,6]. The possibility of separating this singularity is apparently connected with the possibility of observing the discontinuity, since the first terms, at least in the Landau theory, yielded no discontinuity [1-4]. In order for the discontinuity not to become smeared out because of the finite carrier lifetime, the temperature addition to the dispersion law should exceed the uncertainty in the energy, $\Lambda_{ik}u_{ik} \gg \hbar/\tau$. The contribution from the last term in (2) will be largest if the phase transition is accompanied by an increase in the fluctuations of those degrees of freedom which make no contribution to the scattering at the transition temperature. When speaking of the singularity of σ_{ik}, we have assumed that the temperature is sufficiently high to disregard the contribution of the electrons to the thermodynamic quantities, including thermal expansion. On the other hand, if the electronic contribution is appreciable, then u_{ik} is itself a functional of the electronic distribution function. Our analysis pertains only to the case of a transition not accompanied by an appreciable change in the topology of the Fermi surface, the entire change of which is assumed to be connected with thermal expansion.

2. The indicated anomalies should become strongly manifest in ferromagnetic conductors. Let us consider first the anomalous resistance of a ferromagnetic metal. The dispersion law of the conduction electron is of the form

$$\epsilon_{\pm} = \epsilon_0(\vec{p}) + \Lambda_{ik}(\vec{p})u_{ik}(T) \pm \beta M/M_0.$$

Here β is the (s-d) or (s-f) exchange integral responsible for the magnetization, and M/M_0 is the relative magnetization. Neglecting for simplicity the magnetization, we find, as before in (1), that $\Delta\sigma_{ik} = \Sigma_{ik\ell m}u_{\ell m}(T)$. Let us determine $u_{\ell m}(T)$, for which purpose we consider the free energy of a deformed magnetic (without the electronic contribution) [7]

$$F(T, M, u_{ik}) = F_0 + \frac{1}{2}\lambda_{ik\ell m}u_{\ell m} - \Phi M^2 - \Phi_{ik}M^2u_{ik}.$$

In addition to the energy of the elastic deformations (the term responsible for the normal part of the thermal expansion has been omitted), we take into account the exchange-interaction energy $-\Phi M^2$ and the change in this energy upon deformation, due to the dependence of the exchange integrals on the interatomic distances. From $\partial F/\partial u_{ik} = 0$ it follows that $u_{ik}(T) = \lambda^{-1}_{ik\ell m}\Phi_{\ell m}M^2(T)$. We note that the electron dispersion law takes the form (see also [4]):

$$\epsilon_{\pm} = \epsilon_0 + \alpha M^2 \pm \beta M/M_0, \quad \alpha(\vec{p}) = \Lambda_{ik}(\vec{p})\lambda^{-1}_{ik\ell m}\Phi_{\ell m}. \tag{3}$$

The anomalous conductivity $\Delta\sigma_{ik}$ is proportional to the square of the magnetization, $\Delta\sigma_{ik} = \Sigma_{ik}M^2(T)$, where $\Sigma_{ik} = \Sigma_{ik\ell m}\lambda^{-1}_{\ell mst}\Phi_{st}$. For estimating purposes we assume that $\Lambda_{ik} \sim \epsilon_0$ and $\lambda_{ik\ell m} \sim \rho s^2$ (ρ - density, s - speed of sound) and $\Phi_{\ell m} \sim I/M_0^2$ (I - exchange integral responsible for the ferromagnetism and M_0 - saturation magnetization), with $I \sim NkT_c$, where N is the number of magnetic electrons per unit volume and T_c is the Curie temperature. Hence

$$\frac{\Delta\sigma}{\sigma_0} \sim \frac{NkT_c}{\rho s^2}\frac{M^2(T)}{M_0^2}. \tag{4}$$

When $\rho s^2 \sim 10^{-11}$, $N \sim 3 \times 10^{22}$, and $T_c \sim 3 \times 10^2$, the estimate (4) yields the rather small value $\Delta\sigma/\sigma \sim 10^{-2}$. Actually, the variation of M^2 in metals is apparently governed in most cases by other mechanisms (the Mott mechanism, the influence of magnetization, etc [8]). A greater effect may be produced by the anomaly of thermal expansion in ferromagnetic semiconductors, in which the anomalies of the resistance become manifest, in particular, in a change of the slope of the plots of $-\ln\sigma$ vs. $1/T$ at the Curie point. This change is related by Irkhin and Turov to the dependence of the width of the forbidden band on the magnetization, due to the magnetization of the carriers by the magnetic electrons [8]. Similar anomalies should result also from the crystal deformation connected with the anomalous thermal expansion. Indeed, the width of the forbidden band is $\Delta E_{\pm} = \Delta E_0 + \alpha M^2 \pm \beta M/M_0$, where β is the magnetization exchange integral, and αM^2 is given by formula (3) and represents the deformation change in the bottom of the conduction band relative to the edge of the valence band. We then have for the conductivity, in accord with [8]

$$\sigma = A\left\{c_-\exp\left(-\frac{\Delta E_-}{2kT}\right) + c_+\exp\left(-\frac{\Delta E_+}{2kT}\right)\right\},$$

where A and c are slowly varying functions of the temperature, so that when $T > T_c$ $(c_+ + c_-$

$= 1, \Delta E_+ = \Delta E_- = \Delta E_0)$

$$\ln\sigma = \ln A - \Delta E_0/2kT,$$

and when $T \ll T_c$

$$\ln\sigma = \ln Ac_- - (\Delta E_0 + \alpha M_0^2 + \beta)/2kT.$$

Let us compare the term αM_0^2 with β. By virtue of the estimate (4), the "deformation" contribution to the change in the slope of $\ln\sigma$ will exceed the contribution due to magnetization when $NkT_c/\rho s^2 > \beta/\Delta E_0$. To this end it is sufficient to have $\beta/\sqrt{\Delta E_0 kT_c} < 0.1$ if the foregoing numerical estimates are used.

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PION ELECTROPRODUCTION AND AXIAL CURRENT

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We derive in the present note, using the algebra of single-time commutators, a generalization of the Kroll-Ruderman theorem [1] to include the case of electroproduction of pions at momentum transfers $\sim\mu^2$.

The amplitude for the production of a π^+ meson on a proton by a virtual γ quantum can be represented in the form

$$T_\mu = i\int dx\, \exp(iqx)(\Box - \mu^2)\langle p'|\theta(x_0)[j_\mu(0), \varphi(x)]|p\rangle. \tag{1}$$

Here j_μ is the electromagnetic current, φ the meson field operator, and p, p', and q the momenta of the proton, neutron, and meson, respectively.