The results allow us to propose the occurrence of many changes in the topology of the Fermi surface of cadmium, i.e., the occurrence of the electronic transitions predicted by I. M. Lifshitz [7].

In conclusion the authors consider it their pleasant duty to thank Professor L. F. Vereshchagin for interest in the work, and Professor I. M. Lifshitz and A. F. Barabanov for a discussion of the results.

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NOTE CONCERNING APPLICATIONS OF THE HYPOTHESIS OF PARTIALLY-CONSERVED AXIAL-VECTOR CURRENT

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Many papers have been written during the past few months on new consequences of the Gell-Mann chiral SU(3) x SU(3) algebra [1]. In order to obtain results that can be directly compared with experiment, the following two additional hypotheses were made:

A. The divergence of the axial-vector current is proportional to the field

$$\partial^\mu J_\mu^A(x) = -iC\varphi(x), \qquad (A)$$

where $J_\mu^A(x)$ is an axial-vector current with $\Delta S = 0$, and $\varphi(x)$ is the renormalized Heisenberg field of the pion (this is the PCAC hypothesis).

B. It is proposed that certain amplitudes are slowly varying functions of the pion 4-momentum k in the region between $k = 0$ and $k = \mu$, where μ is the pion mass.

The purpose of this note is to point out that the operator equation A (PCAC) has in fact never been used in the cited papers. We shall show that if hypothesis B is valid, then the predictions of these papers are valid, regardless of whether the hypothesis A is valid or not (or even whether this hypotehsis is approximately valid). On the other hand, if hypothesis B is invalid, then the predictions are likewise invalid, even if hypothesis A is correct.

Hypothesis A (PCAC) was used in the cited papers in the following manner. The matrix element of a certain local operator $\theta(x)$ was considered, in the form $\langle A^{in}|\theta(x)|\pi(\vec{k})B^{out}\rangle$, where $|\pi(\vec{k})B^{out}\rangle$ is an arbitrary state containing a pion with momentum $\vec{k}$. The Lehmann-Symanzik-Zimmermann reduction formula makes it possible to represent it in the form

$$(2k^0)^{\frac{1}{2}} \langle A^{in}|\mathcal{O}(x)|\pi(\vec{k})B^{out}\rangle = i\int d^4y \, \exp(iky)\overrightarrow{(-\partial^2 + \mu^2)}_y \langle A^{in}|T(\mathcal{O}(x)\varphi(y)|B^{out}\rangle, \qquad (1)$$

where $\varphi(y)$ is the renormalized Heisenberg field of the pion, and $k^0 = (\vec{k}^2 + \mu^2)^{\frac{1}{2}}$. Application of hypothesis A yields

$$(2k^0)^{\frac{1}{2}} \langle A^{in}|\mathcal{O}(x)|\pi(\vec{k})B^{out}\rangle = -\frac{1}{c}\int d^4y \, \exp(iky)\overrightarrow{(-\partial^2 + \mu^2)}_y \langle A^{in}|T(\mathcal{O}(x), \partial^\mu J^A_\mu(y)|B^{out}\rangle. \quad (2)$$

If we assume here (hypothesis B) that the expression in the right part of (2) is a slowly varying function of k, then it can be approximated with the aid of its value at k = 0. The latter can be calculated by using the current algebra. Therefore PCAC is used here only to obtain Eq. (2).

We shall show that Eq. (2) is generally valid without any additional hypotheses of the PCAC type if the constant C is chosen equal to $\mu^2 f_\pi$, where f_π differs from zero and is defined by the equation

$$\langle 0|\partial^\mu J^A_\mu(0)|\pi(\vec{k})\rangle = -i\mu^2 f_\pi/(2k^0)^{\frac{1}{2}}, \qquad (3)$$

and is the amplitude of the $\pi \to \mu + \nu$ decay.

Before demonstrating this, let us present a brief derivation of (1), in order to emphasize that the only property of the field $\varphi(x)$ used to obtain (1) is that it satisfies an asymptotic condition defined below.

We define an operator

$$a_k(t) \equiv \int d^3x \, \exp(-ikx)\, i\overleftrightarrow{\partial}_0 \varphi(x), \qquad (4)$$

where

$$i\overleftrightarrow{\partial}_0 \equiv i\overrightarrow{\partial}_0 - i\overleftarrow{\partial}_0$$

and

$$k^0 = (\vec{k}^2 + \mu^2)^{\frac{1}{2}}.$$

Then the asymptotic condition denotes that

$$\lim_{t\to-\infty} a_k(t) = a_k^{out},$$

and (5)

$$\lim_{t\to+\infty} a_k(t) = a_k^{in},$$

where a_k^{in+} and a_k^{out+} are operators of creation of physical single-pion states with momentum $\vec{k}$.

For example

$$|\pi(\vec{k})B^{out}\rangle = a_k^{out+}|B^{out}\rangle,$$

and (6)

$$\langle A^{in}|a_k^{in+} = 0,$$

inasmuch as $\langle A^{in}|$ dies not contain a pion in a state with momentum $\vec{k}$.

In order to show that (1) is a direct consequence of the asymptotic condition (5), we need only use the identity

$$a_k^+(t_1)\mathcal{O}(y) - \mathcal{O}(y)a_k^+(t_2) = -i\int_{t_2}^{t_1} dx^0 \int d^3x \frac{\exp(ikx)}{(2k^0)^{\frac{1}{2}}} \overrightarrow{(-\partial^2 + \mu^2)_x} (\mathcal{O}(y)\varphi(x)) \tag{7}$$

at $t_1 > y^0 > t_2$. This identity is obtained by integrating by parts and using the definition (4) of the operator $a_k(t)$.

Let us consider the matrix element of both parts of (7) between the states $\langle A^{in}|$ and $|B^{out}\rangle$, and let $t_1 \to +\infty$ and $t_2 \to -\infty$. Using the asymptotic condition (5) and the properties (6) of the operators a_k^{in+} and a_k^{out+}, we arrive at Eq. (1). We note that from the foregoing derivation it follows that the correctness of Eq. (1) does not depend on the form of the equations of motion or commutation relations satisfied by φ.

We consider now an arbitrary local operator $\tilde{\varphi}(x)$, for which the matrix element $\langle 0|\tilde{\varphi}(x)|\pi(\vec{k})\rangle$ does not vanish. It then follows from relativistic invariance that

$$\langle 0|\tilde{\varphi}(0)|\pi(\vec{k})\rangle = \gamma/(2k^0)^{\frac{1}{2}}, \tag{8}$$

where γ is a certain nonvanishing constant.

Then, if we define

$$\tilde{a}_k(t) \equiv \frac{1}{\gamma}\int d^3x(2k^0)^{-\frac{1}{2}}\exp(-ikx)\, i\overleftrightarrow{\partial_0}\tilde{\varphi}(x) \tag{4'}$$

then, as shown by Haag [2],

$$\lim_{t\to-\infty} \tilde{a}_k(t) = a_k^{out}$$

$$\lim_{t\to+\infty} \tilde{a}_k(t) = a_k^{in}. \tag{5'}$$

Thus, asymptotically the operator $a_k(t)^+$, just like $a_k^+(t)$, is the operator for the creation of a physical single-pion state with momentum $\vec{k}$. In other words, the operator $\tilde{\varphi}(x)/\gamma$ satisfies the asymptotic condition.

Using (4') and (5') in place of (4) and (5), we repeat the derivation of (1) given above, making the substitution $\varphi(x) \to \tilde{\varphi}(x)/\gamma$ and $a_k(t) \to \tilde{a}_k(t)$.

We then obtain in lieu of (1) the following equation for $\langle A^{in}|\mathcal{O}(x)|\pi(\vec{k})B^{out}\rangle$:

$$(2k^0)^{\frac{1}{2}} \langle A^{in}|\mathcal{O}(x)\pi(\vec{k})B^{out}\rangle = \frac{i}{\gamma}\int d^4y\, \exp(iky)\overrightarrow{(-\partial^2 + \mu^2)_y} \langle A^{in}|T(\mathcal{O}(x)\tilde{\varphi}(y)|B^{out}\rangle, \tag{1'}$$

where $k^0 = (\vec{k}^2 + \mu^2)^{\frac{1}{2}}$. Of course, the right sides of (1) and (1') are equal only when $k^2 = -\mu^2$.

It is convenient (and by virtue of (3) possible) to choose

$$\tilde{\varphi}(x) = \partial^\mu J_\mu^A(x). \tag{9}$$

Comparing (3) with (8), we obtain for the indicated choice of $\tilde{\varphi}(x)$

$$\gamma = -i\mu^2 f_\pi, \tag{10}$$

and (1') coincides with (2) with $C = \mu^2 f_\pi$. Thus, Eq. (2) with $C = \mu^2 f_\pi$ is exact and always valid. There is no need to introduce the renormalized Heisenberg pion field $\varphi(x)$ in terms of (1), and then remove it with the aid of PCAC. This remark is quite trivial and is well known [3]. Nonetheless, we wish to point out that the hypothesis that the divergence of the axial-vector current is proportional to the pion-field operator is referred to in the literature with increasing frequency. It is not actually needed at all to derive the result (2).

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GENERATION OF ELECTROMAGNETIC OSCILLATIONS IN AN OPEN RESONATOR

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The use of open resonators is quite promising for the generation and amplification of electromagnetic oscillations. It makes it possible to improve appreciably the frequency selection, a fact especially important for the short millimeter and submillimeter bands. A self-exciting generator using an open resonator can be realized by passing a straight-line