ASYMPTOTIC BEHAVIOR OF THE ELECTRIC CONDUCTIVITY IN DISORDERED STRUCTURES

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We consider a system of centers that are randomly placed at points $\vec{R}_i$, with random electron energy level ε_i at the i-th center. At low temperatures $T \ll E_0$ (E_0 is the average depth of the level in the forbidden band for the mobility) the electron motion is via jumps between centers with absorption of a phonon or of an energy quantum of the external electromagnetic field. For this case, Mott has shown [1] that at very low temperatures $T \ll \bar{\varepsilon}$ ($\bar{\varepsilon}$ is the average distance between levels of the neighboring centers near the Fermi energy ε_F) the electric conductivity is

$$\sigma \sim \exp(-\text{const}\, T^{-1/4}), \tag{1}$$

and the coefficient of absorption of electromagnetic waves in the region of very low frequencies ω is

$$a \sim \omega^2 (\ln \omega)^4 . \tag{2}$$

Mott does not indicate the region of applicability of formulas (1) and (2).

In the present paper we find the upper limit ω_0 of the applicability of formula (2). We also show that at higher frequencies another formula holds for the dependence of the electric conductivity on the frequency (see formula (4)). In addition, we indicate the lower limit T_0 of applicability of formula (1) and find the $\sigma(T)$ dependence at $T \ll T_0$.

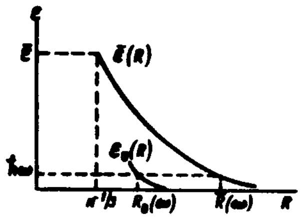
Let us consider first the electric conductivity in an alternating field with frequency $\hbar\omega \ll \bar{\varepsilon}$. Following Mott, we assume the average distance $\bar{\varepsilon}(V)$ between successive energy levels in the volume V to be $(N_F V)^{-1}$, where N_F is the level density per unit volume at the Fermi energy. The jumps occur predominantly between successive levels, since the jumps in which a level is skipped have exponentially small probability. Equating the field-quantum energy to $(N_F V)^{-1}$, we obtain the distance over which the jump takes place

$$\bar{R}(\omega) = \left(\frac{4\pi N_F \hbar \omega}{3}\right)^{-1/3} . \tag{3}$$

The transition probability is proportional to the overlap factor $\exp(-2\alpha R)$. It follows therefore that the electric conductivity depends on the frequency in the following manner:

$$\sigma \sim \exp(-\text{const}\, \omega^{-1/3}). \tag{4}$$

Let us establish the lower limit of applicability of formulas (1) and (4). We carry out the analysis for the frequency dependence (4), but the result is perfectly valid also for (1). As shown by Lifshitz [2], the difference between levels of two centers spaced by a distance R cannot be smaller than the energy of the exchange splitting $\varepsilon_0(R) = IR\exp(-\alpha R)$ (I is the exchange integral and α is the reciprocal Bohr radius). Plots of the dependences of the average distance between levels $\bar{\varepsilon}(R)$ and of the minimal distance $\varepsilon_0(R)$ are shown in the figure. We see that at low frequencies the length of the jump cannot be smaller than $R_0(\omega) = (1/\omega)\ln(1/\alpha\hbar\omega)$. The overlap factor $\exp(-2\alpha R)$ decreases rapidly with increasing R. From this point of view, the most convenient are jumps over the minimum possible distances. But the probability of encountering centers with an energy difference much smaller than the average value $\bar{\varepsilon}$ is very



Dependence of the average energy $\bar{\varepsilon}(R)$ and the minimum energy $\varepsilon_0(R)$ on the distance R between centers.

small. To estimate the probability W_0 of finding the minimum possible level difference $\varepsilon_0(R)$ at a given R, we use the formula $W_0 \sim N_F \varepsilon_0(R)$.

Thus, transitions to energy levels with minimum spacing make a contribution $\sim N_F \exp(-3\alpha R)$. Comparing this quantity with the contribution of the jumps to $\bar{\varepsilon}(R)$, we observe that they become equal if the following condition is satisfied:

$$3R_0(\omega) = 2\bar{R}(\omega). \tag{5}$$

We denote by ω_0 the smaller of the two roots of (5).

At frequencies $\hbar\omega_0 \ll \hbar\omega \ll \bar{\varepsilon}$, formula (4) is valid, and the temperature dependence of (1) takes place at temperatures $T_0 \ll T \ll \bar{\varepsilon}$, where $T_0 = \hbar\omega_0$.

We write the expression for the jumps of the electric conductivity in an alternating field at T = 0 in the form

$$\sigma(\omega) = \frac{2\pi e^2}{m^2\omega}\int_0^\infty dR\, p^2(R, \omega)\, F(\hbar\omega, R), \tag{6}$$

where

$$F(\hbar\omega, R) = \sum_{i,j} \delta(R - R_{ij})\,\delta(\hbar\omega + \varepsilon_i - \varepsilon_j)\, n_i(1 - n_j)$$

$$n_i = \left[\exp\left(\frac{\varepsilon_i - \varepsilon_F}{T}\right) + 1\right]^{-1}$$

and the matrix element of the momentum $p(R, \omega)$, taken between wave functions localized on centers spaced at a distance R apart with energy difference $\hbar\omega$, is proportional to $R^2\exp(-\alpha R)$.

At frequencies $T_0 \ll \hbar\omega \ll \bar{\varepsilon}$, the function $F(\hbar\omega, R)$ is proportional to the distribution function $W_F(\varepsilon)$ of the distances between neighboring levels near the Fermi energy, for a sample in the form of a sphere with radius R. Namely,

$$F(\hbar\omega, R) = n\frac{d}{dR} W_F(\hbar\omega - \bar{\varepsilon}(R)),$$

where n is the concentration of the centers. Pokrovskii [3] has shown that the function $W_F(\varepsilon)$ has a sharp maximum at an energy ε equal to the average distance between neighboring levels. Integration with respect to R is eliminated, owing to the δ-like character of $W_F(\varepsilon)$, and as a result we get (4).

At lower frequencies $\hbar\omega \ll T_0$, the function $F(\hbar\omega, R)$ is proportional to

$$\frac{d}{dR}\left[1 - \exp\left(-\frac{4\pi R^3 N_F \hbar\omega}{3}\right)\right] = 4\pi R^2 \hbar\omega N_F$$

with $R \geq R_0(\omega)$. Substituting this expression for $F(\hbar\omega, R)$ in (6) and integrating, we obtain Mott's formula (2).

The static conductivity $\sigma(T)$ can be written in the form

$$\sigma(T) = \frac{2\pi e^2}{m^2 T} \int_0^\infty dR \int_0^\sim d\epsilon\, \rho^2(\epsilon, R)\, V^2(\epsilon, R)\, e^{-\epsilon/T} F(\epsilon, R) \tag{7}$$

where

$$V(\epsilon, R) = \frac{E_1 R \epsilon^{3/2}}{2\sqrt{3\rho}\,\hbar^4 c^5},$$

E_1 is the constant of the deformation potential, ρ is the density of matter, and c is the speed of sound.

When $T_0 << T << \bar{\varepsilon}$, the function $F(\varepsilon, R)$ has, as before, a sharp maximum at $\varepsilon = \bar{\varepsilon}(R)$; taking the integral with respect to R by the saddle-point method we obtain (1).

At $T << T_0$ we have $F(\varepsilon, R) \sim 4\pi R^2 N_F \varepsilon$, and

$$\sigma \sim \frac{1}{T} \int d\epsilon\, \epsilon^4 e^{-\epsilon/T} \sim T^4 . \tag{8}$$

We see thus that the conductivity in either a constant or an alternating field, at low temperatures and low frequencies, goes over from an exponential decrease to a power-law decrease. The experimental data of Austin [4] and Austin and Mott [5] for the temperature dependence $\sigma(T)$ offer evidence in favor of this statement.

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ACTIVATION ENERGY OF JUMP CONDUCTIVITY

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At low semiconductor impurity concentrations, the wave functions of the localized electrons have a weak overlap. The low-temperature conductivity of such a semiconductor is connected with jumps of the electron from occupied to unoccupied impurity states [1]. For concreteness, we shall speak of an n-type semiconductor, in which there are N_D donors and N_A acceptors. At low temperatures all the N_A acceptors are negatively charged, there are electrons on $(N_D - N_A)$ donors, and the remaining N_A donors are empty. It has been established experimentally that the dependence of the jump conductivity on the temperature T has an activation character. In the present paper we find the activation energy of the jump conductivity ε_3 in the case of small compensation