integrating, we obtain Mott's formula (2).

The static conductivity $\sigma(T)$ can be written in the form

$$\sigma(T) = \frac{2\pi e^2}{m^2 T} \int_0^\infty dR \int_0^\infty d\epsilon \, p^2(\epsilon, R) V^2(\epsilon, R) e^{-\epsilon/T} F(\epsilon, R) \qquad (7)$$

where

$$V(\epsilon, R) = \frac{E_1 R \epsilon^{3/2}}{2\sqrt{3\rho} \hbar^4 c^5},$$

E_1 is the constant of the deformation potential, ρ is the density of matter, and c is the speed of sound.

When $T_0 \ll T \ll \bar{\epsilon}$, the function $F(\epsilon, R)$ has, as before, a sharp maximum at $\epsilon = \bar{\epsilon}(R)$; taking the integral with respect to R by the saddle-point method we obtain (1).

At $T \ll T_0$ we have $F(\epsilon, R) \sim 4\pi R^2 N_F \epsilon$, and

$$\sigma \sim \frac{1}{T} \int d\epsilon \, \epsilon^4 \, e^{-\epsilon/T} \sim T^4 . \qquad (8)$$

We see thus that the conductivity in either a constant or an alternating field, at low temperatures and low frequencies, goes over from an exponential decrease to a power-law decrease. The experimental data of Austin [4] and Austin and Mott [5] for the temperature dependence $\sigma(T)$ offer evidence in favor of this statement.

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ACTIVATION ENERGY OF JUMP CONDUCTIVITY

B.I. Shklovskii, A.L. Efros, and I.Ya. Yanchev
A.F. Ioffe Physico-technical Institute, USSR Academy of Sciences

At low semiconductor impurity concentrations, the wave functions of the localized electrons have a weak overlap. The low-temperature conductivity of such a semiconductor is connected with jumps of the electron from occupied to unoccupied impurity states [1]. For concreteness, we shall speak of an n-type semiconductor, in which there are N_D donors and N_A acceptors. At low temperatures all the N_A acceptors are negatively charged, there are electrons on $(N_D - N_A)$ donors, and the remaining N_A donors are empty. It has been established experimentally that the dependence of the jump conductivity on the temperature T has an activation character. In the present paper we find the activation energy of the jump conductivity ϵ_3 in the case of small compensation

$K = N_A/N_D \ll 1$. In the case of small compensation, most donors are filled with electrons, and only a few of them are free. The unfilled donor states can be regarded as carriers: when the electron goes over from a filled donor to a free one, a positive vacancy moves over the lattice. At low temperatures, all the vacancies are in states with minimum energy, i.e., in the direct vicinity of the negatively charged acceptors [1, 2]. Therefore the Fermi level turns out to be higher than the energy of the isolated donor by an amount on the order of the energy of the Coulomb interaction of the acceptor with the nearest donor. At $K \ll 1$, the acceptors lie far from each other, and therefore the jump of the vacancy directly from one acceptor to the other is impossible. In order to move over the crystal, the vacancy must become detached from the acceptor and jump over the donors that are located far from the acceptors. The levels of such donors are shifted very little by the potential of the charge acceptors and donors. Therefore the activation energy is determined by the difference between the Fermi level and the energy of the isolated impurity.

To find the Fermi level at $K \ll 1$ and $T = 0$, we neglect the interaction between donors located in the immediate vicinity of one acceptor and other acceptors. Let us find the number of ionized donors that can be located in the vicinity of one acceptor at a certain positive value of μ. The Fermi level, like all energies in the present paper, is reckoned upward from the level of the isolated impurity. First, it may turn out that there is not a single ionized donor near the given acceptor (0-complex). This can occur if the donors have accidentally produced a rarefaction around the acceptor, so that the increment made to the electron energy ε by the negative acceptor at the donor closest to it does not exceed μ. Second, there may be one ionized donor near the acceptor (1-complex). Third, there may be two ionized donors near the acceptor (2-complex). Such a complex, for example, can arise if two donors are located close to the acceptor, but on opposite sides. It can be easily verified that at positive μ there can be no three ionized donors near one acceptor.

The chemical potential μ is determined by the neutrality condition, i.e., by the condition that the concentrations of the 0- and 2-complexes $N_0(\mu)$ and $N_2(\mu)$ be equal. The concentration $N_0(\mu)$ can be readily determined. Indeed, the 0-complex exists when there is not a single donor at a distance $r_\mu = e^2/\kappa\mu$ from the fixed acceptor (here e is the electron charge and κ is the dielectric constant). The probability of such an event is equal to $\exp[(4\pi/3)N_D r_\mu^3]$. (We assume that there is no correlation in the locations of the impurities and we use therefore a Poisson distribution.) Thus,

$$N_0(\mu) = N_A \exp\left(-\frac{4\pi}{3}\frac{e^6 N_D}{\kappa^3 \mu^3}\right).$$

An exact calculation of $N_2(\mu)$ is difficult. We shall therefore use in place of the concentration $N_2(\mu)$ the two quantities $N_2^>(\mu)$ and $N_2^<(\mu)$ which are close to it and straggle it. ($N_2^>(\mu) > N_2(\mu) > N_2^<(\mu)$.) We shall show that this "fork" is so narrow that the corresponding uncertainty in the value of μ is smaller than 1%. The quantity $N_2^>(\mu)$ is the total concentration of the donor pairs, at which in the ionized state the two increments to the energy levels ε_1 and ε_2 exceed μ. (We shall call this a μ-pair.) The presence of a μ-pair near an acceptor denotes almost always the existence of a 2-complex. Exceptions are cases when there exists near the acceptor, besides the μ pair, also one close donor, so that a 1-complex with this donor is energywise more favored than a 2-complex with the pair. Another reason why $N_2^>(\mu)$ exceeds $N_2(\mu)$ slightly is that near the acceptor, generally speaking, there may be several μ pairs of donors. In this case only the energywise most favored μ pair will be really ionized, and there will be only one 2-complex. Let $\vec{r}_1$ and $\vec{r}_2$ be the radius vectors drawn from the acceptor to the first and second donors, respectively. (These donors, generally speaking, may not be the nearest neighbors

of the acceptors.) In the case of a random dis-
tribution of the impurities we have

$$N_2^>(\mu) = N_A N_D^2 \int d^3 r_1 \int d^3 r_2 \, \theta\{\epsilon_1(r_1, r_2) - \mu\} \theta\{\epsilon_2(r_1, r_2) - \mu\}. \quad (1)$$

$$|r_2| \geqslant |r_1|$$

Here ϵ_1 and ϵ_2 are the increments to the energies
of the electrons at the donors as a result of the
potential of the acceptor and the second donor

$$\epsilon_1 = \frac{e^2}{\kappa|r_1|} - \frac{e^2}{\kappa|r_1 - r_2|}; \quad \epsilon_2 = \frac{e^2}{\kappa|r_2|} - \frac{e^2}{\kappa|r_1-r_2|}. \quad (2)$$

We assume that the characteristic $\vec{r}_1$, $\vec{r}_2$, and
$|\vec{r}_1 - \vec{r}_2|$ are large compared with the Bohr radius
of the electron at the donor, so that the level
shift can be regarded as classical. $\theta(x) = 1$ at
$x > 0$ and $\theta(x) = 0$ at $x < 0$. The second quantity
calculated by us, $N_2^<(\mu)$, is the concentration of
the μ pairs produced by two donors nearest to
their acceptor. We shall prove that $N_2^<(\mu) < N_2(\mu)$.
Indeed, the presence of a μ pair of two nearest
donors is only a sufficient but not necessary con-
dition for the existence of a 2-complex. A 2-com-

ϵ_3, meV

$10^{-5} N^{1/3} cm^{-1}$

Dependence of the acti-
vation energy ϵ_3 of the
jump resistance on the
concentration of the
main impurities for the
least doped samples in-
vestigated in [4, 5]:
o – n-Ge, • – p-Ge.

plex can exist, for example, if the two nearest donors do not make a μ pair at
all, but the μ pair is made up of the first and third donors. This situation
is possible when the first and second nearest donors are on one side of the
acceptor, and the third on the other side. Thus, $N_2^<(\mu) < N_2(\mu)$. For $N_2^<(\mu)$ we
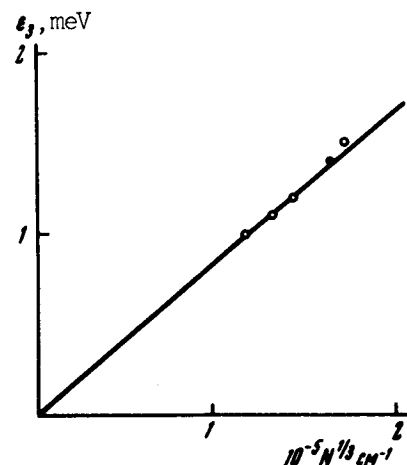have

$$N_2^<(\mu) = N_A N_D^2 \int d^3 r_1 \int d^3 r_2 \, e^{-\frac{4\pi}{3} N_D r_2^3} \theta\{\epsilon_1(r_1, r_2) - \mu\} \theta\{\epsilon_2(r_1, r_2) - \mu\}.$$

$$\qquad\qquad\qquad (3)$$

$$|r_2| \geqslant |r_1|$$

Solving with the aid of a computer the equations $N_2^<(\mu) = N_0(\mu)$ and $N_2^>(\mu) = N_0(\mu)$
we obtain values of μ that differ by less than 1%. With the same accuracy, we
have

$$\epsilon_3 = \mu = \alpha \frac{e^2 N_D^{1/3}}{\kappa}; \qquad \alpha = 0.99. \quad (4)$$

Miller and Abrahams [3] found that $\alpha = 1.61$. This significant difference is
due to the fact that the authors of [3] did not take 0- and 2-complexes into
account. The experimental value of α, obtained with the aid of the figure, is
$\alpha_{exp} = 0.93 \pm 0.01$, which is in satisfactory agreement with (4).

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