SPIN WAVES IN NONFERROMAGNETIC METALS WITH OPEN FERMI SURFACES

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The spin waves in nonferromagnetic metals, predicted by the theory of a degenerate electron liquid [1, 2], were observed experimentally [3, 4] in alkali metals, the Fermi surfaces of which are nearly spherical. At the same time, such spin waves can exist also in metals with sharply anisotropic Fermi surfaces. Several theoretical premises for spin waves in such metals were formulated in [5 - 7], and in concrete applications of the theory of [5] principal attention was paid to closed Fermi surfaces. In the present communication we present results pertaining to the case of unclosed surfaces, leading to the presence of open electron trajectories in momentum space, and revealing essential features of spin waves in such metals.

Assume that among the electron trajectories

$$\epsilon = \epsilon_0 = \text{const}\ , \qquad p_z = \text{const}\ , \tag{1}$$

where the z axis is oriented along the constant magnetic field B, there are both closed and open ones. Then in accordance with [5], the dispersion equation of the spin waves with frequency ω and wave vector $\vec{k}$ can be written in the form

$$\left(\frac{\beta}{1+\beta} + \frac{i}{\omega\tau}\right) X(\omega, \mathbf{k}) = 1 \tag{2}$$

Here unlike in [5]

$$X(\omega, \mathbf{k}) = X^{o}(\omega, \mathbf{k}) + X^{c}(\omega, \mathbf{k}), \tag{3}$$

where X^c is the contribution of the closed trajectories, determined in [5], and the contribution of the open trajectories is given by the expression

$$X^{o}(\omega, \mathbf{k}) = \left[\int \frac{dS}{|\mathbf{v}(\mathbf{p})|}\right]^{-1} \int_{S_o} \frac{dS}{|\mathbf{v}(\mathbf{p})|} \frac{\omega}{\omega \pm \Omega_0 - \mathbf{k}\mathbf{v} + \frac{i}{\tau} + \frac{i}{\tau_2}}, \tag{4}$$

where S_o denotes that the integration is carried out over the surface of the open trajectories, $\vec{v}(\vec{p})$ is the velocity of the electron on the surface, τ_1 and τ_2 are the relaxation times of the momentum in the spin of the electron, respectively

$$\beta = \frac{2\psi}{(2\pi\hbar)^3} \int \frac{dS}{|\mathbf{v}(\mathbf{p})|} \tag{5}$$

ψ is a constant characterizing the correlation of the electrons, and $\Omega_0 = 2\mu_0 B/(1 + \beta)\hbar \equiv \omega_s/(1 + \beta)$ is the characteristic frequency of the spin resonance, with ω_s the usual Bloch resonance frequency due to the spin reversal of the conduction electron.

Bearing in mind the expression obtained in [5] for X^c, let us discuss the consequences that follow from the dispersion equation [2]. Assuming the relaxation time to be sufficiently large, we neglect the wave dissipation. We discuss first the long-wave limit, when we have from (2)

$$\omega = \pm\omega_s \left\{1 + \frac{1}{\beta\Omega_0^2}\left[<(\mathbf{k}\mathbf{v})^2>_0 + k_z^2 < v_{z0}^2 >_c - \right.\right.$$
$$\left.\left. - \sum_{m \neq c} < \frac{|\mathbf{k}\mathbf{v}_m|^2}{m\left(\frac{\Omega}{\omega_3}\right)^2\left(1 + \frac{1}{\beta}\right)^2 - 1} >_c \right]\right\} \tag{6}$$

here the averaging over the closed and open Fermi surfaces is determined respectively by the formulas

$$< F >_H = \left[\int \frac{dS}{|v|}\right]^{-1} \int_{S_0} \frac{dS}{|v|} F; \quad < F >_c = \left[\int \frac{dp_z}{\Omega}\right]^{-1} \int_{S_c} \frac{dp_z}{\Omega} F$$

Ω is the cyclotron frequency of the Larmor rotation of the conduction electron, and for closed trajectories we can expand the velocity vector in a Fourier series

$$\mathbf{v} = \sum_{m=-\infty}^{+\infty} \mathbf{v}_m e^{im\phi} .$$

The unique character of the contribution of the open trajectories is connected with the occurrence of a term that does not depend on the orientation of the magnetic field relative to the direction of the anisotropic Fermi surface. We emphasize that here, too, a qualitative difference is observed between the spin waves and the cyclotron waves. Namely, whereas the cyclotron waves become forbidden in the presence of open trajectories, this is not the case for spin waves.

Nevertheless, the open trajectories do impose a certain hindrance on the spin waves. This hindrance, however, differs qualitatively from the hindrance of the cyclotron waves. In order to demonstrate this, we turn to the case of propagation of spin waves in a direction perpendicular to the constant magnetic field. As shown in [5], in the case of a closed anisotropic Fermi surface, the dependence of the Larmor frequency on the momentum p_z leads to the presence of forbidden bands, outside of which the spin waves are possible for all wavelengths, and, in particular, in the limit of short waves, for wavelengths much shorter than the radius of the gyroscopic rotation of the electron. The presence of open trajectories leads to a limitation on the possible wavelengths or, accordingly, on the values of the wave vectors k of the spin waves. Indeed, by directing the x axis along the vector k, we can easily see from expression (4) that, upon satisfaction of the equality

$$k v_{x,\,max} = \omega \pm \Omega_0 , \tag{7}$$

where $v_{x,max}$ is the maximum value of the x-projection of the electron velocity, resonant coherent absorption of the wave by electrons is possible upon reversal of their spin. Such an absorption, obviously, has as its analog the inverse Cerenkov effect, which leads to Landau damping. Formula (7) makes it possible to express the frequency in terms of the wave vector and makes it possible, in conjunction with the dispersion equation (2), to find the value of the limiting wave vector and of the frequency corresponding to it. We illustrate the foregoing using as an example open trajectories produced by intersection of the cylindrical Fermi surface with the planes p_z perpendicular to the cylinder axis. Then (4) takes the form

$$\chi^{\circ} = \frac{\omega}{2\pi} \int_0^{2\pi} \frac{d\phi}{\omega \pm \Omega_0 - kv\cos\phi} . \quad (8)$$

It follows therefore that the limiting wave vector is equal to Ω_0/v. When the wave vector approaches such a limiting value, the frequency of the spin wave decreases, tending to zero like

$$\omega = (\Omega_0 - kv)\left\{1 + \frac{\beta^2}{2(1+\beta)^2}\frac{\Omega_0 - kv}{kv}\right\}.$$

We note that the vanishing of the frequency at the limiting wave vector is an indication that there can exist a spatially-periodic paramagnetic structure which, unlike the structure considered in [8], is periodic in a direction transverse to the constant magnetic field.

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