NONLINEAR VARIATION OF THE PHASE VELOCITY AND STABILIZATION OF PLASMA OSCILLATIONS

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Usually in the study of plasma turbulence one starts from the concept of a source and sink of plasma oscillations. The level of the stationary turbulence is determined in this case from the condition of balance between the source and the sink. In particular, in the theory developed by Kadomtsev for ion-acoustic turbulence in a plasma with current [1], the source is Cerenkov excitation of waves by moving electrons, and the sink is nonlinear interaction of the waves and particles, leading to a transfer of energy from the waves to the ions of the plasma (nonlinear Landau damping).

We show in the present paper that in principle there is possible another mechanism of establishing stationary turbulence in a plasma. Namely, owing to nonlinear effects, a change takes place in the phase velocity of the plasma wave, and this change is proportional to the level of the turbulent oscillations. At sufficiently high levels of turbulent oscillations, the Cerenkov condition ceases to be satisfied for all waves that can propagate in the plasma. As a result, the plasma goes over into a stationary state characterized by a very high level of turbulent oscillations - the state of stationary turbulence.

By way of an example, let us consider ion-acoustic oscillations in a strongly magnetized collisionless plasma with hot electrons, through which a cold beam of low density passes along the magnetic field. The dispersion equation for the low-frequency oscillations has in this case the form

$$1 + (ak)^{-2} - \Omega_i^2 \omega^{-2} \cos^2\theta - \Omega_b^2 (\omega - ku\cos\theta)^{-2} \cos^2\theta + 4\pi\tilde{\kappa} = 0. \quad (1)$$

Here a is the electron Debye radius, $\Omega_{i,b}$ the Langmuir frequency for the plasma ions and beam particles, u the directional beam velocity, θ the angle between the wave vector $\vec{k}$ and the magnetic field, and $\tilde{\kappa}$ the nonlinear correction to the electric susceptibility of the plasma (in the first approximation in the fluctuation correlator), due to the random waves. To determine $\tilde{\kappa}$ we must solve the kinetic equations for the electrons and ions accurate to terms cubic in the wave amplitude, separate terms having the same structure as the linear electric susceptibility, and finally to carry out averaging over the random phases of the waves. Being interested in magnetosonic oscillations, we can confine ourselves to calculation of the longitudinal (relative to the corresponding wave vectors) electric susceptibility and to assume that $v_i \ll \omega/k \ll v_e$ ($v_{e,i}$ is the thermal velocity of the electrons and ions).

We present here the final expression for $\tilde{\kappa}$ in the case of a plasma in which magnetosonic oscillations can propagate, with wavelengths not exceeding a certain maximum value $2\pi k_m^{-1}$. Such a situation takes place in a bounded plasma (when k_m is equal in order of magnitude to the reciprocal dimensions of the system), or in the case of a plasma with nonvanishing low collision frequency (when k_m is determined from the condition $\omega_{k_m} \sim \tau^{-1}$). Assuming for concreteness that $1 \gg (ak_m)^4 \gg T_i/T_e$ ($T_{e,i}$ is the temperature of the electrons and ions for the plasma), we obtain

$$\tilde{\kappa} = \frac{\cos\theta\,(\Delta k)^3 < E^2 >_k}{12\sqrt{2}\pi n_0 \sqrt{T_e T_i}}, \quad \cos\theta > 0, \quad (2)$$

where $\langle E^2 \rangle_k$ is the spatial Fourier component of the correlator of the electric field in the magnetosonic region, $\Delta k \sim (\Omega_b/\Omega_i)^{2/3} k_m (ak_m)^{-2}$ is the bandwidth of the waves excited by the beam, and n_0 is the plasma density. (If $\cos\theta < 0$, $\tilde{\kappa}$ is proportional to the fluctuation level in the non-turbulent region, and is therefore small.)

Using (1), we can easily find the correction to the phase velocity of the wave

$$\Delta v_\phi = -V_s \frac{\cos\theta\,(\Delta k)^3 \langle E^2 \rangle_k (ak_m)^2}{6\sqrt{2}\, n_0 \sqrt{T_e T_i}\,(1 + a^2 k_m^2)^{3/2}}, \quad \cos\theta > 0 \tag{3}$$

(V_s is the velocity of the magnetic sound).

With increasing level of the turbulent fluctuations, the quantity Δv_ϕ, remaining negative, increases in absolute magnitude and ultimately reaches a value

$$\Delta v_m = \frac{V_s}{\sqrt{1 + a^2 k_m^2}} - u + \frac{V_s (ak_m)^2 \Delta k}{2k_m}. \tag{4}$$

The condition of Cerenkov excitation then ceases to be satisfied for all the waves capable of propagating in the plasma, so that further growth of the waves stops. As a result, the plasma goes over into a stationary state characterized by a fluctuation level

$$\langle E^2 \rangle_k = \frac{\Delta v_m}{V_s} \frac{6\sqrt{2}\, n_0 \sqrt{T_e T_i}\,(1 + a^2 k_m^2)^{3/2}}{\cos\theta\,(\Delta k)^3 (ak_m)^2}, \quad \cos\theta > \frac{1}{\tau k_m V_s} \tag{5}$$

(at $\cos\theta < (\tau k_m V_s)^{-1}$, the fluctuations are not turbulent and are characterized by a much lower level).

The ratio of the wave energy to the particle energy in such a plasma is

$$\frac{U_w}{U_e} \sim \frac{\Delta v_m}{V_s} \sqrt{\frac{T_i}{T_e}} \left(\frac{\Omega_b}{\Omega_i}\right)^{-4/3}. \tag{6}$$

We see that the stabilization of the plasma instability can set in already at low values of the wave energy (compared with the particle energy), i.e., still under conditions of the so-called weak turbulence. We note that the criterion for the applicability of the theory is not the condition $U_W \ll U_e$, but, generally speaking, the weaker condition $\tilde{\kappa} < (ak)^{-2}$.

The foregoing formulas pertain to the case of a beam with not too low a density, $n_b/n_0 > (m_e/m_i)^{3/2}(m_b/m_i)$, when it is possible to disregard the wave attenuation due to their interaction with the plasma particles. It can be shown that in the case of a beam of low density, the mechanism under consideration will lead to stabilization of the instability.

Such a situation can take place, for example, if a potassium-ion beam of density $n_b \sim 10^{11} - 10^{12}$ cm^{-3} passes through a hydrogen plasma with a density $n_0 \sim 10^{14}$ cm^{-3} and particle temperatures $T_e \sim 10^6$ deg, and $T_i \sim 10^4$ deg, placed in a magnetic field $B_0 \sim 5 \times 10^5$ G. At a beam velocity $u \sim 10^7$ cm/sec, the parameter $\Delta v_m/V_s$, which characterizes the supercriticality, should be of the order of 10^{-2} (the frequency of the ion collisions is $\tau^{-1} \sim 10^{10}$ sec^{-1}, corresponding to collisions with neutral particles whose density is $n \sim 0.01n_0$).

[1] B.B. Kadomtsev, Plasma Turbulence, in: Voprosy teorii plazmy (Problems of Plasma Theory), M.A. Leontovich, editor, Atomizdat, 1964, No. 4, p. 188.

RELAXATION OF μ^+-MESON SPIN IN SUBSTANCES WITH SATURATED BONDS

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According to the presently accepted theory [1 - 3], the depolarization of a μ^+ meson in matter is due to the formation of a hydrogenlike muonium atom. The produced muonium can have a spin I = 0 and I = 1. In the case of polarized μ^+ mesons, both these states are produced with equal probability. The production of muonium with I = 0 leads to depolarization of the μ^+ meson within a time $t = 1/\omega_0 \approx 3.6 \times 10^{-11}$ sec, where ω_0 is the frequency of the hyperfine splitting of the ground state of the muonium. When muonium with I = 1 is produced, the μ^+ meson is not depolarized. Owing to the interaction with the medium, the muonium electron can charge the spin direction, which leads to transitions $I = 1 \rightleftarrows I = 0$, which lead to further depolarization of the μ^+ meson. The muonium process of depolarization terminates when muonium enters into a chemical bond with the molecules of the medium. The two parameters not calculated in the theory, namely the lifetime τ of the muonium and the frequency ν of the depolarization of the muonium electrons, determine the residual polarization P_{res} of the μ^+ meson in the substance at the instant of termination of the muonium stage. It is obvious, that P_{res} increases with decreasing τ. The dependence of P_{res} on ν has a more complicated form. At $\nu = 0$, the polarization P(t) decreases rapidly within a time t_0 to $P_{res} = 1/2$, and subsequently remains constant. With increasing ν at a given τ, the polarization P_{res} first decreases, reaches a minimum at $\nu \approx \omega_0$, and then begins to increase and tends to unity at $\nu >> \omega_0$ [3].

We have investigated the depolarization of the μ^+ meson in saturated hydrocarbons (hexane, heptane, octane) and in methyl alcohol - substances with saturated bonds. It can be assumed that the time τ that muonium enters into a chemical reaction with the molecules of these substances is relatively long. The large lifetime of the muonium, $\tau > 1/\omega_0$, at low frequencies $\nu << \omega_0$ should lead to a complete depolarization of the μ^+ meson in the state with I = 0, i.e., to $P_{res} < 1/2$. The values $\tau < 1/\omega_0$ and $\nu << \omega_0$ should lead, in addition, to an appreciable dependence of the residual polarization P_{res} of the μ^+ meson on the transverse magnetic field $H_\perp$ [3]. The $P_{res}(H_\perp)$ dependence is due to the fast "muonium" precession of the spin of the μ^+ meson during the lifetime τ of the muonium. The spins of the individual μ^+ mesons "turn" in this case through different angles, leading to a decrease of the residual polarization P_{res}.

Experiment did not confirm these assumptions (see the table). It was found that the residual polarization of the μ^+ meson in the investigated substances exceeds the value $P_{res} = 1/2$ and does not depend on $H_\perp$. Such a result does not agree with the muonium theory of depolarization of the μ^+ meson either at $\tau < 1/\omega_0 = 3.6 \times 10^{-11}$ sec or at high frequencies $\nu > \omega_0$ [3].