Attention is also called to the fact that when the solution concentration is increased, an increase is observed in the scatter in the values of the width of the emission spectrum from flash to flash. The width of the spectrum varied from flash to flash in the range from $\Delta\nu = 2.5\ cm^{-1}$ to $\Delta\nu = 13\ cm^{-1}$ for a laser with a cell thickness $\ell = 1$ mm and from $\Delta\nu = 2\ cm^{-1}$ to $\Delta\nu = 60\ cm^{-1}$ for $\ell = 3\ \mu$. The increase in the scatter of the width of the spectra agrees with the concepts developed in [7], where the observed scatter is connected with separation by means of a passive shutter with $\tau_{rel} > \Delta t$ of short-duration realizations of the noise picture of the radiation. The slower the duration of the separated groups of USP, the larger should be the value of the scatter.

The demonstrated possibility of reducing the relaxation time of a shutter makes it possible to make substantial progress towards the solution of the problem of increasing the power of existing solid-state lasers, which is presently hindered considerably by the lack of rapidly-relaxing dyes.

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HYPOTHESIS OF GRAVITATIONAL RADIATION OF A RAPIDLY ROTATING NEUTRON STAR

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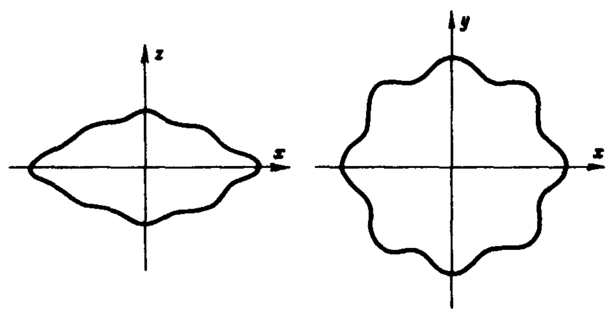
It is assumed in this article that rapidly-rotating neutron stars can assume tesseral equilibrium shapes (see the figure), and should at the same time emit powerful pulses of gravitational radiation on going over from one tesseral shape to another. It is possible that these pulses were registered by J. Weber a few years ago [1, 2].

Assume we have a self-gravitating liquid of mass M and constant density ρ, rotating as a whole with angular velocity Ω about a certain axis z passing through the center of gravity of the liquid. This rotating drop assumes an equilibrium form such that the following condition is satisfied on its surface [3]

$$\Phi(x, y, z) - \frac{\Omega^2}{2}(x^2 + y^2) = \text{const}. \quad (1)$$



where Φ is the gravitational potential and (x, y, z) are Cartesian coordinates. The shapes of a rotating liquid most thoroughly investigated and known at present are: a) oblate ellipsoid of revolution (the so-called Maclaurin ellipsoid), b) triaxial ellipsoid (the Jacobi ellipsoid). The M and J ellipsoids have known regions of stability with respect to the angular momentum [3]. At large values of the angular momentum of the liquid, other equilibrium shapes are also possible [4]. The existence of these shapes is demonstrated by perturbation-theory methods, in which the "unperturbed" shapes are assumed to be the M and J ellipsoids. If the "unperturbed" shape is assumed to be the M ellipsoid, then an elliptical system of coordinates (ξ, μ, ϕ) is introduced [5]. On the surface of the ellipsoid $\xi = f(a^2 - f^2)^{-1/2} \equiv \xi_0$ (a - major semiaxis, f - minor semiaxis of the ellipsoid), whereas for the "perturbed" surface we have $\xi = \xi_0 + \Delta\xi(\theta\phi)$; $|\Delta\xi| << \xi_0$; $\mu = \cos\theta$

$$\Delta\xi(\theta\phi) = (\mu^2 + \xi_0^2)^{-1} \sum_{\ell m} B_{\ell m} Y_{\ell m}(\theta\phi) \tag{2}$$

$Y_{\ell m}(\theta\phi)$ are spherical functions. $\Delta\xi(\theta\phi)$ must satisfy the condition (1) on the surface of the liquid. It turns out that if the parameter of the Maclaurin ellipsoid ξ_0 satisfies the equation

$$\xi_0\left[1 - \xi_0\left(\frac{\pi}{2} - \operatorname{arc\,tg}\xi_0\right)\right] - i(-1)^m \frac{(\ell - m)!}{(\ell + m)!} P_\ell^m(i\xi_0) Q_\ell^m(i\xi_0) = 0 \tag{3}$$

$$(m > 0;\ \ell \neq 0;\ 1;\ \ell + m - \text{even})$$

then only two terms in the series (2) differ from zero, namely $B_{\ell m}Y_{\ell m}(\theta\phi)$ and its complex conjugate. $P_\ell^m(z)$ and $Q_\ell^m(z)$ in (3) are associated Legendre functions of the first and second kind. The ellipsoid whose ξ_0 coincides with one of the roots of Eq. (3) is called the branching ellipsoid. For $\ell >> 1$ and $m \sim \ell$ we have $\xi_0 \sim 1/\sqrt{2\ell}$ and the number of roots is $\sim\ell^2/4$. The deformation of the surface of the ellipsoid vanishes on a number of meridians and a number of parallels, and transforms the ellipsoid into a tesseral equilibrium shape (see the figure). An investigation of the stability of these equilibrium shapes has shown that when $|\Delta\xi| << \xi_0$ (the humps and the troughs are small) the shapes are unstable, with the exception of the case $\ell = m = 2$. It is interesting to note that this case $\ell = m = 2$ corresponds to the appearance of a J ellipsoid. This confirms also the value of the root of the equation (3), $\xi_0 \simeq 0.717$. That is to say, the J ellipsoid can be regarded as a "perturbed" M ellipsoid, and not as an independent equilibrium figure (so long as $|\Delta\xi| << \xi_0$). The main assumption in this paper consists in the following: the tesseral equilibrium shapes constructed on the basis of the M ellipsoid are stable at sufficiently large deformation ($|\Delta\xi| \gtrsim \xi_0$). Apparently there exist equilibrium shapes that are superpositions of different tesseral shapes, but we make no assumptions concerning their stability.

Let us apply this information to a rotating neutron star during the first stages of its existence, when it can have the necessary angular momentum. Allowance for the compressibility of the matter in the neutron star will apparently not change the picture radically, for the following reason. It is shown in [6] that at an equation of state $p = k\rho^{1+(1/n)}$ with $n < 1$, a rotating drop can assume the form of a J ellipsoid ($\ell = m = 2$) before matter begins to flow away from the equator of the M ellipsoid. For neutron stars, as follows from [7], there exists a mass region where $n < 1$. We assume that the compressibility of the matter in the neutron star (at $n < 1$) will not prevent the existence also of higher tesseral equilibrium shapes ($\ell >> 1$, $m >> 1$). The solid envelope of the neutron star apparently cannot prevent the formation of tesseral shapes. The observed jumplike changes in the periods of pulsars, $\Delta\Omega/\Omega \sim 10^{-6}$ (star quakes) are due to this solid envelope (see [8]). The smallness of this value indicates that the crust is still not a serious obstacle to the establishment of an equilibrium shape of the liquid drop. It can be shown that if the condition $2\Omega a/c \lesssim 1$ is satisfied (c is the speed of light), a rotating neutron star of tesseral shape with sufficiently large ℓ and m does not emit gravitational waves. On the other hand, electromagnetic waves are emitted by the stars and their density is $I = (2\Omega^4/3c^3)\mu_\perp^2$ ($\mu_\perp$ is the component of the magnetic field of the star perpendicular to the axis of revolution). The star in this case loses its angular momentum slowly and leaves the region of stability of the given tesseral shape. The duration of this process depends on the width of the stability zone, which we do not know at present. For the numerical example given below, we assume that this time is $\sim 10^6$ sec. In the stability region it loses its ordered shape - gravitational radiation is produced (in the quadrupole approximation, at a frequency $\omega = 2\Omega$), and continues until the star falls into the stability region of another tesseral shape, with other ℓ and m (this process is quite rapid). The intensity of the gravitational radiation is $(-dE/dt) \sim (G\omega^6 M^2 a^4 \beta^2/5c^5)$; (G is the gravitational constant and β is the fraction of stellar mass contained in the humps). The displacement of the ends of the Weber cylinders under the influence of a gravitational-radiation pulse is [9]

$$\eta \sim 2\frac{\ell_0}{r}\frac{G\omega^2}{c^4}\left(\frac{\omega}{\delta}\right) M a^2 \beta$$

where $\ell_0 \sim 10^2$ cm, $r \sim 2.4 \times 10^{22}$ cm is the distance to the wave sources (we choose them to the center of the galaxy), and δ is the half-width of the spectral density of the radiation. We specify the parameters $\ell \sim 40$, $\rho \sim 4 \times 10^{14}$ g/cm^3, and $a \sim 4 \times 10^6$ cm, and then $\xi_0 \sim 0.11$. The angular velocity and the mass of the star are estimated from the formula for the branching ellipsoid. $\Omega^2/2\pi\rho G \sim 0.13$; $\Omega \sim 4.7 \times 10^3$ sec^{-1}, $I \sim 40^{43} - 10^{44}$ erg/sec, $\omega = 2\Omega \sim 0.94 \times 10^4$ sec^{-1}, $M \sim 4\pi\rho a^3\xi_0/3 \sim 11.2 \times 10^{33}$ g $\sim 5.6M_\odot$; $-dE/dt \sim 1.2\beta^2 10^{58}$ erg/sec, and if $\beta \sim 1/60$ then $-dE/dt \sim 4 \times 10^{54}$ erg/sec. The total energy of the gravitational radiation of one star is $\sim 2 \times 10^{54}$ erg. If the number of pulses is $\sim 2 \times 10^2$, then the energy of one pulse is $\sim 10^{52}$ erg and the pulse duration is 3×10^{-3} sec, corresponding to $(\omega/\delta) \sim 5$; we then obtain $\eta \sim 10^{-15}$ cm, and the radiation is in the frequency range (1500 ± 300) Hz. Thus, for generation of $\sim 2 \times 10^2$ pulses with $\eta \sim 10^{-15}$ cm there should be produced at the center of the galaxy ~1 rapidly rotating neutron star per year. At smaller β, the δ band decreases and η increases. The fundamental frequency changes slowly as the pulses are radiated, and only a fraction of the pulses falls in the Weber band (1660 Hz). It is possible to choose the parameters such that relativistic effects, capable of increasing the value of η, become significant. Weber [1] estimated the registered pulses of gravitational radiation at $\eta \sim 2 \times 10^4$ cm (~80 pulses/year), and in a subsequent paper [2], at weaker signals, η is approximately one order of magnitude smaller (~400 pulses/year). During the slowing down process, the star with $M > 2.4M_\odot$ [10] collapses and at $M < 2.4M_\odot$ it turns into an ordinary pulsar. The difficulties of explaining Weber's results with the aid of presently known mechanisms for the radiation of gravitational waves are discussed in the review [11].

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NONLINEAR EFFECTS IN THE PROPAGATION OF MONOCHROMATIC VLF WAVES (HELICONS) IN THE MAGNETOSPHERE

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In this paper we discuss certain nonlinear effects arising when monochromatic electronic circularly polarized waves in the VLF band ("whistlers," helicons) propagate along the force lines of the geomagnetic field. For concreteness, we have in mind the region of the internal electron belt ($L \simeq 2.5$), where the plasma consists of two components: cold, $N \simeq 10^3$ cm^{-3} with temperature $\sim 1 - 10$ eV, and hot, $n \simeq 10^{-3}$ cm^{-3}. The latter has an anisotropic distribution of the electron velocity ($T_\perp/T_\parallel \simeq 2$ and $T_\perp \simeq 100$ keV) [1]. We shall assume that the amplitude of the monochromatic signal sent from the earth into the magnetosphere and arriving at the magnetically-conjugate point greatly exceeds the amplitude of the noise resulting from the instability of the distribution function of the hot plasma. On entering the amplification zone (which is localized in the equatorial region), the initial wave will first be amplified with an increment determined by the linear theory [2]

$$\gamma_L = \sqrt{\pi}\,\frac{n}{N}\,\frac{\omega_c^2\left(1-\frac{\omega_0}{\omega_c}\right)^2 v_{T_\perp}^2}{k_0 v_{T_\parallel}^3}\exp\left\{-\left(\frac{\omega_0-\omega_c}{k_0 v_{T_\parallel}}\right)^2\right\}\left[1-\frac{T_\parallel}{T_\perp}-\frac{\omega_0}{\omega_c}\right], \tag{1}$$

where ω_c is the cyclotron frequency of the electrons and ω_0 is the frequency of the initial wave.

After a time on the order of one period of the oscillations of the particles captured by the wave

$$\tau = (k_0 \omega_c v_{T_\perp} h/H)^{-1/2} \tag{2}$$

where h/H is the ratio of the amplitude of the alternating magnetic field in the wave to the constant geomagnetic field, a nonlinear stage sets in. The amplification of the wave then stops and its amplitude, after several oscillations with period (2), reaches a stationary value h_0, which does not differ greatly from the initial value h if $\gamma_L\tau \ll 1$. (This inequality was satisfied in the experiments discussed in the present paper (e.g., [3, 4]).

At the same time, the distribution function is greatly distorted in the resonant region of velocity space, as a result of which satellites are excited. The increments of the latter, as can be readily shown, are determined by the expression

$$\gamma = \gamma_L \Phi(\Omega\tau), \tag{3}$$

where

$$\Omega = \left(1 - \frac{v_\phi}{v_g}\right)(\omega - \omega_0), \tag{4}$$