

NONLINEAR EFFECTS IN THE PROPAGATION OF MONOCHROMATIC VLF WAVES (HELICONS) IN THE MAGNETOSPHERE

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In this paper we discuss certain nonlinear effects arising when monochromatic electronic circularly polarized waves in the VLF band ("whistlers," helicons) propagate along the force lines of the geomagnetic field. For concreteness, we have in mind the region of the internal electron belt ($L \approx 2.5$), where the plasma consists of two components: cold, $N \approx 10^3 \text{ cm}^{-3}$ with temperature $\sim 1 - 10 \text{ eV}$, and hot, $n \approx 10^{-3} \text{ cm}^{-3}$. The latter has an anisotropic distribution of the electron velocity ($T_{\perp}/T_{\parallel} \approx 2$ and $T_{\perp} \approx 100 \text{ keV}$) [1]. We shall assume that the amplitude of the monochromatic signal sent from the earth into the magnetosphere and arriving at the magnetically-conjugate point greatly exceeds the amplitude of the noise resulting from the instability of the distribution function of the hot plasma. On entering the amplification zone (which is localized in the equatorial region), the initial wave will first be amplified with an increment determined by the linear theory [2]

$$\gamma_L = \sqrt{\pi} \frac{n}{N} \frac{\omega_c^2 \left(1 - \frac{\omega_0}{\omega_c}\right)^2 v_{T_{\perp}}^2}{k_0 v_{T_{\parallel}}^3} \exp \left\{ - \left(\frac{\omega_0 - \omega_c}{k_0 v_{T_{\parallel}}} \right)^2 \right\} \left[1 - \frac{T_{\parallel}}{T_{\perp}} - \frac{\omega_0}{\omega_c} \right], \quad (1)$$

where ω_c is the cyclotron frequency of the electrons and ω_0 is the frequency of the initial wave.

After a time on the order of one period of the oscillations of the particles captured by the wave

$$\tau = (k_0 \omega_c v_{T_{\perp}} h/H)^{-1/2} \quad (2)$$

where h/H is the ratio of the amplitude of the alternating magnetic field in the wave to the constant geomagnetic field, a nonlinear stage sets in. The amplification of the wave then stops and its amplitude, after several oscillations with period (2), reaches a stationary value h_0 , which does not differ greatly from the initial value h if $\gamma_L \tau \ll 1$. (This inequality was satisfied in the experiments discussed in the present paper (e.g., [3, 4]).

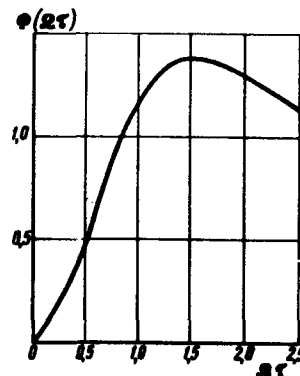
At the same time, the distribution function is greatly distorted in the resonant region of velocity space, as a result of which satellites are excited. The increments of the latter, as can be readily shown, are determined by the expression

$$\gamma = \gamma_L \Phi(\Omega \tau), \quad (3)$$

where

$$\Omega = \left(1 - \frac{v_{\phi}}{v_g} \right) (\omega - \omega_0), \quad (4)$$

ω is the frequency of the satellite, and v_ϕ and v_g are the phase and group velocities of the wave, respectively. The function $\Phi(\Omega\tau)$ (which is even) is shown in the figure. The derivation of (4) and an analytic expression for the function $\Phi(\Omega\tau)$ will be published in a more detailed communication¹⁾ (for plasma waves without a magnetic field, the corresponding theory has been developed in [5]).



If we disregard the case of very weak dispersion, when $v_\phi \rightarrow v_g$ (this limiting case takes place when $\omega_0 \rightarrow \omega_c/2$), then $\Omega \sim (\omega - \omega_0)$. It is then seen from the figure that the excitation increment of the satellites (red and violet) has a maximum at $|\Omega\tau| \approx 1.5$, i.e., $|\omega - \omega_0| \sim 1/\tau$. Recognizing that the line width of the fundamental wave, due to the oscillations of the captured particles, is of the same order, and also the possibility of insufficient resolution capability, we can state, at least, that experimental observation of the initial wave should reveal broadening of the spectrum by an amount on the order of $1/\tau$. According to [2], this broadening is proportional to the square root of the amplitude. This result can be verified experimentally²⁾.

During the next stage there should occur an interaction between the initial wave and the satellites, leading to an energy exchange between them. The characteristic time T of energy transfer can be estimated in the following manner: $T \geq 1/\gamma$, where γ is the growth increment of the satellites (3). Since (see the figure) $\max \Phi \sim 1$, it follows that $\gamma \approx \gamma_L$ and consequently $T \geq 1/\gamma_L$. The energy transfer from one wave to the satellites and back should lead to amplitude and frequency modulation of the wave, with a characteristic time on the order of T . Since $\gamma_L \tau \ll 1$, we have $T \gg \tau$.

For the parameters given above for the plasma in the propagation region, we get from (1) $\gamma_L \approx 10 \text{ sec}^{-1}$. Thus, modulation effects can be observed in those cases when the duration of the transmitted signal is at least several times larger than 0.1 sec. We note that the foregoing considerations agree with the experimental results reported in [4]³⁾.

In conclusion, the authors take this opportunity to thank Ya.I. Likhter, O.A. Molchanov, and V.M. Chmyrev for a discussion of the results.

¹⁾In the derivation of formulas (2) and (3) it was assumed that

$$\frac{h}{H} \frac{\omega_c^3}{k_0^3 v_{Te}^3} \ll 1.$$

This condition was satisfied in the experiments of [3, 4].

²⁾At parameters corresponding to the conditions of [3], we have $\tau \approx 0.02$ sec. A broadening on the order of 10^2 Hz is then expected for the spectrum.

³⁾We did not touch upon here the question of the origin of the discrete trigger radiation observed in a number of investigations, particularly in [3, 4], which is connected with but not equivalent to the problem considered here.

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HIGH-FREQUENCY ELECTROMAGNETIC FIELD IN A PLASMA CAVITY

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We calculate the spectrum of the radiation whose pressure is balanced by the pressure of a surrounding plasma with arbitrary density profile.

1. Volkov [1] considered a one-dimensional "well" in a plasma, filled with monochromatic radiation of frequency ω , the pressure of which is balanced by the plasma pressure (Fig. 1). The wave equation

$$\Delta E - \frac{\partial^2}{\partial t^2} E = \frac{\partial^2}{\partial x^2} E + k^2(x) E = 0, \quad k(x) = \frac{1}{c} \sqrt{\omega^2 - \omega_0^2(x)}, \quad (1)$$

where $\omega_0(x) = (4\pi e^2 N(x)/m)^{1/2}$ leads in this case to a unique solution for the field, $E_y(x, t) = E_\omega(x) \sin(\omega t + \phi)$, which in turn is regarded as connected with the plasma density $N = N_e = N_i$ by a Boltzmann distribution [2, 3] (with $T_+ = T_e + T_i = \text{const}$)

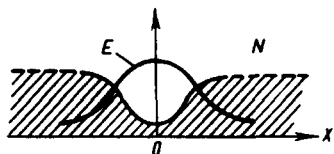
$$N(x) = N(\infty) e^{-U/T_+} \quad \text{where} \quad U = \frac{e^2}{4m\omega^2} E_\omega^2(x). \quad (2)$$

It will be shown below that if we forego the monochromaticity condition and assume that a large number of natural oscillations are excited in the well, then this problem admits of a general solution for an arbitrary symmetrical monotonic profile of the density $N = N(|x|)$.

2. In the geometrical-optics approximation, the eigenfrequencies ω_n are determined from the quantization condition $\pi(n + 1/2) = \int k_0(x) dx$, which when differentiated with respect to n yields

$$1 = \frac{\partial}{\partial n} \frac{1}{\pi} \int_{-x_0}^{x_0} k_n(x) dx = \int_{-x_0}^{x_0} \frac{\omega_n \partial \omega_0 / \partial n}{\pi c^2 k_n(x)} dx, \quad (3)$$

which can be regarded as the condition for the normalization of the eigenfunctions



$$\psi_n(x) = \frac{\text{const}}{\sqrt{k(x)}} \sin \left(\int_{-x_0}^x k(x) dx + \frac{\pi}{4} \right) \quad (4)$$