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HIGH-FREQUENCY ELECTROMAGNETIC FIELD IN A PLASMA CAVITY

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We calculate the spectrum of the radiation whose pressure is balanced by the pressure of a surrounding plasma with arbitrary density profile.

1. Volkov [1] considered a one-dimensional "well" in a plasma, filled with monochromatic radiation of frequency ω, the pressure of which is balanced by the plasma pressure (Fig. 1). The wave equation

$$\Delta E - \frac{\partial^2}{c^2 \partial t^2}\hat{\epsilon} E = \frac{\partial^2}{\partial x^2} E + k^2(x) E = 0, \quad k(x) = \frac{1}{c}\sqrt{\omega^2 - \omega_0^2(x)}, \tag{1}$$

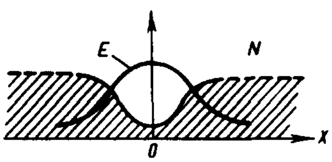
where $\omega_0(x) = (4\pi e^2 N(x)/m)^{1/2}$ leads in this case to a unique solution for the field, $E_y(x, t) = E_\omega(x)\sin(\omega t + \phi)$, which in turn is regarded as connected with the plasma density $N = N_e = N_i$ by a Boltzmann distribution [2, 3] (with $T_+ = T_e + T_i = \text{const}$)

$$N(x) = N(\infty) e^{-U/T_+} \text{ where } U = \frac{e^2}{4m\omega^2} E_\omega^2(x). \tag{2}$$

It will be shown below that if we forego the monochromaticity condition and assume that a large number of natural oscillations are excited in the well, then this problem admits of a general solution for an arbitrary symmetrical monotonic profile of the density $N = N(|x|)$.

2. In the geometrical-optics approximation, the eigenfrequencies ω_n are determined from the quantization condition $\pi(n + 1/2) = \int k_0(x)dx$, which when differentiated with respect to n yields

$$1 = \frac{\partial}{\partial n}\frac{1}{\pi}\int_{-x_0}^{x_0} k_n(x) dx = \int_{-x_0}^{x_0} \frac{\omega_n \partial\omega_0/\partial n}{\pi c^2 k_n(x)} dx, \tag{3}$$



which can be regarded as the condition for the normalization of the eigenfunctions

$$\psi_n(x) = \frac{\text{const}}{\sqrt{k(x)}}\sin\left(\int_{-x_0}^{x} k(x)\, dx + \frac{\pi}{4}\right) \tag{4}$$

which are solutions of (1) in the WKB approximation.

Assuming

$$E_y(x, t) = \sum_n E_n(x) \sin(\omega_n t + \phi_n), \; E_n(x) = A_n \psi_n(x), \tag{5}$$

where ϕ_n and A_n are the phase and amplitude of the n-th harmonic we have for the potential (2)

$$U(x) = \frac{e^2}{4m} \sum_n \frac{1}{\omega_n^2} E_n^2(x) \cong \frac{e^2}{4m} \int \frac{1}{\omega_n^2} A_n^2 \psi_n^2(x) dn =$$

$$= \frac{e^2}{4m} \int \frac{A_n^2}{\omega_n^2} \frac{\omega_n \partial\omega_n/\partial n}{\pi c^2 k_n(x)} dn = \frac{e^2}{4\pi mc} \int_{\omega_o(x)}^{\omega_p(\infty)} \frac{A_n^2 \, d\omega_n}{\omega_n \sqrt{\omega_n^2 - \omega_o^2(x)}}. \tag{6}$$

We have replaced here the sum by an integral and took into account the normalization (3) of the function $\psi_n^2(x)$. Yet from (2) we have

$$U(x) = T_+ \ln \frac{1}{g(x)}, \text{where } g(x) = N(x)/N(\infty). \tag{7}$$

Equating (6) and (7) and introducing a new integration variable $s = \omega_n^2/\omega_0^2(\infty)$, we obtain the Abel integral equation

$$\ln \frac{1}{g} = \int_g^1 \frac{f(s)\,ds}{\sqrt{1-g}}, \quad \text{where } f(s) = \frac{e^2 c\omega_o(\infty)}{8\pi mcT_+} \frac{A_n^2}{a_n^2}. \tag{8}$$

Its solution is the function

$$f(s) = \frac{2}{\pi\sqrt{s}} \operatorname{arc\,tg} \sqrt{\frac{1-s}{s}}, \tag{9}$$

which makes it possible to determine the amplitudes A_n and by the same token all the characteristics, and particularly the spectrum, of the radiation balancing the given profile g(x).

It can be shown that the inverse problem of determining the balancing density profile from the specified spectrum w_ω of the total radiation energy

$$w = \int w_\omega d\omega, \qquad w_\omega = \frac{1}{8\pi} A_n^2 \frac{dn}{d\omega_n} \tag{10}$$

can be solved in similar fashion, and the spectrum w_ω should satisfy definite conditions, which will not be discussed here.

3. Using, as above, the geometrical-optics approximation, let us consider a three-dimensional spherically-symmetrical "well" with a density profile N = N(r). The motion of the transverse quanta in the inhomogeneous plasma is described by the equations

$$\dot{\mathbf{r}} = \frac{\partial\Omega}{\partial\mathbf{p}} = \frac{\mathbf{p}}{\mu}, \quad \dot{\mathbf{p}} = -\frac{\partial H}{\partial\mathbf{r}} = -\frac{a}{\omega_k} \nabla N(r) = \mathbf{F}_k, \tag{11}$$

where

$$p = \hbar k, \quad H = \hbar\omega_k, \quad \omega_k = \sqrt{\omega_0^2(r) + c^2 k^2}, \quad \mu = H/c^2, \quad a = 2\pi\hbar e^2/m.$$

If Φ_k is the distribution function of the quanta, normalized such that $\int \Phi_k d\vec{k}$ is equal to the number of quanta per unit volume, then the potential (2) and the local energy density of the radiation are determined by the integrals

$$U(r) = a\int \frac{\Phi_k}{\omega_k} dk, \quad w(r) = \frac{1}{4\pi} E^2 = \int \hbar\omega_k \Phi_k dk \tag{12}$$

and at equilibrium (see (2)) we have $U = T_+ \ln(1/g(r))$, where $g(r) = N(r)/N(\infty)$ ($g(\infty) = 1$). It can be shown that in this case there is no solution with locally-isotropic radiation, when $\Phi_k = f(|\vec{k}|)$, and the simplest radiation is the one consisting of quanta moving only on circular orbits around the center of the well. Equating the gradient force (11) to the centrifugal force, we have

$$\frac{a}{\omega_k}\frac{\partial N}{\partial z} = \mu\frac{v_\perp^2}{r}, \text{otherwise } c^2 k^2 = \omega_0^2(\infty)\frac{r}{2}\frac{\partial g}{\partial r}. \tag{13}$$

Therefore, the only quanta that can move on a circle of given radius r are those with frequency

$$\omega_k = \sqrt{\omega_0^2(r) + c^2 k_\perp^2} = \omega_0(\infty)\sqrt{g + \frac{r}{2}\frac{\partial g}{\partial r}}, \tag{14}$$

and the quantities w and U are connected by the relation

$$w(r) = \left(2N + r\frac{\partial N}{\partial r}\right)U(r) = T_+ N(\infty)\left(2g + r\frac{\partial g}{\partial r}\right)\ln\frac{1}{g} \tag{15}$$

while the connection of (14) between the frequency and the radius makes it possible to calculate all the spectral characteristics of such an anisotropic radiation captured in a spherical cavity.

4. The last system can be of definite interest in connection with the hypothesis of Dawson and Jones [4], that ball lightning is a spherical cavity with radiation whose pressure is balanced by the outer pressure of the atmosphere. In [4] it was assumed, however, that only several first modes of the oscillations are excited in the cavity (with dimension ∿10 cm), and this raised the question (among many other questions) of how powerful radio emission with wavelengths on the order of 10 cm can arise in thunderstorm discharges. In the author's opinion, the lowering of the characteristic wavelengths to the millimeter or even submillimeter band (such wavelengths could be ascribed to synchrotron radiation of ordinary lightnings) makes it possible to supplement the model [4] with a hypothetical mechanism of formation of ball lightning as a result of radio emission "forced out" from the skin-layer channel of ordinary lightning with subsequent formation of an autonomous cavity screened by a plasma sheath. I plan to discuss these problems in a separate paper.

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MODULATION OF SPECTRUM AND AMPLITUDES OF LOW-FREQUENCY SIGNAL IN THE MAGNETOSPHERE PLASMA

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We discuss in this communication certain results of an experiment on the sounding of the magnetosphere of the earth by powerful low-frequency pulses with carrier frequency f_0 = 15 kHz. Since it is known that such frequencies channel themselves in the magnetosphere plasma along the force lines of the earth's magnetic field [1], the signals are received in a region that is magnetically conjugate to the transmitter. Pulses of duration τ = 200, 400, and 800 msec and repetition period T = 3 sec were transmitted. At practically all times, two signals were received: the waveguide signal passing over the earth's surface in the earth-ionosphere waveguide, along a route $S_1 \approx$ 10,000 km long, with a delay $t_1 \approx$ 32 msec, and the magnetosphere signal, with a delay $t_2 \approx$ 500 - 900 msec, which corresponds at a route length $S_2 \approx$ 30,000 km to an average group refractive index along the trajectory $n_g \approx 5 - 10$. An example of the dynamic spectrum of the received signal is shown in Fig. 1. The axes represent the frequency f and the running time t, and the spectral intensity of each instant of time is represented by the blackening of the paper. We see that unlike the waveguide signal A_1 (τ = 400 msec), the dynamic spectrum broadens in a certain region of the magnetosphere signal A_2, and quasimonochromatic induced (called "trigger") radiation A_3 appears. The maximal broadening of the noise spectrum does not exceed the limits $\Delta f_{n\cdot max} \approx$ 100 - 400 Hz. The frequency of the trigger radiation f_T can either increase or decrease, and sometimes the $f_T(t)$ dependence has several extrema and the radiation continues 100 - 300 msec after the cessation of the primary signal at the frequency f_0, $\Delta f_{T\cdot max} = |f_T - f_0|_{max}$ = 1 - 3 kHz. Trigger radiation was already observed earlier [2] and will not be discussed in detail in this paper.

In our experiment we observed the effect of periodic modulation of a quasinoise spectrum of the magnetosphere signal and the associated amplitude modulation. This phenomenon was first observed by us in 1968 [3], but it became manifest most clearly in the described experiment, when the duration of the transmissions was increased. Figure 2 shows the dynamic spectrum (a) and the amplitude plot (b) of the received signal. The duration of the radiated pulse in this case was τ = 800 msec; the waveguide signal was eliminated from the amplitude plot, since the two signals, as can be seen from the spectral plot, partly overlap owing to the insufficiently large delay of the magnetosphere signal (the slow variation of the carrier frequency, seen in Fig. 2a, is due to instability of the recording apparatus and is not connected with the described effect).

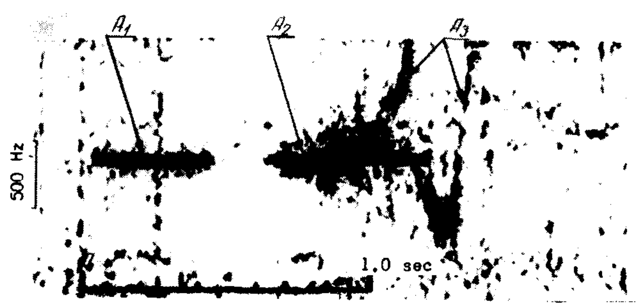

Fig. 1