ELECTROMAGNETIC EXCITATION WITH POLARIZATION OPPOSITE TO THE HELICON IN AN UNCOMPENSATED MAGNETOACTIVE PLASMA

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It is well known that no right-polarized electromagnetic waves can propagate in a single-component plasma situated in a magnetic field when $qR \ll 1$ and $\omega < \Omega$ [1] (ω and q are the frequency and the wave vector, Ω the cyclotron frequency, and R the Larmor radius). The possibility of the propagation of such waves in the region $qR > 1$ and $\omega < \Omega$ is connected only with quantization of the orbital motion of the electrons in a strong magnetic field, which is the cause of the transparency sections in the regions of collisionless damping, leads to the appearance of a series of right-polarized waves [2]. It was customary to assume that a similar situation exists also in a two-component uncompensated plasma, i.e., no low-frequency wave with polarization opposite to that of the helicon can propagate. In the present communication we show that a collective excitation with polarization opposite to that of the helicon and with frequency smaller than Ω propagates in an uncompensated magnetoactive plasma.

The initial frequency of this wave is proportional to the quality $n_1|e_1| - |e_2|n_2$, which vanishes in an equilibrium ionosphere and gas-discharge plasma by virtue of the electroneutrality condition (n - concentration, e - particle charge). In an electron-hole solid-state plasma, however, it need not necessarily vanish, since there exists a lattice to cancel the excess charge. This circumstance points to the possible propagation of a new collective excitation in an ionosphere (and not necessarily only ionosphere) plasma, if the role of the lattice is assumed by heavier particles of a third kind. We confine ourselves in this paper to the case of a magnetoactive solid-state plasma and use a model of a two-component plasma consisting of electrons of mass m_1 and concentration n_1 and of holes of mass m_2 and concentration n_2. Such a plasma is realized, for example, in doped bismuth.

The dispersion equation at $\vec{q} \parallel \vec{H}$, $qR \ll 1$ and $\omega^2 \ll \omega_p^2$ is

$$\omega_{p_1}^2 \frac{\Omega_1}{\Omega_1 \mp \omega} + \omega_{p_2}^2 \frac{\Omega_2}{\Omega_2 \pm \omega} = \omega_{p_1}^2 + \omega_{p_2}^2 + c^2 q^2 . \tag{1}$$

Here $\omega_p = 4\pi n e^2/m$, c is the speed of light, the upper (lower) sign corresponds to left (right) polarized wave, and the index 1 (2) pertains to the electrons (holes).

We solve Eq. (1) without imposing any conditions on the ratio ω/Ω in the case when $q^2 \ll q_k^2 = \delta_1^{-2}(n_1 - n_2)^2 m_1/(n_1 m_2 + n_2 m_1)n_1$, where $\delta = c/\omega_p$. As a result we obtain

$$\omega^{\mp} = \mp \frac{\Omega_1 \Omega_2}{\Omega_1 \omega_{p_2}^2 - \Omega_2 \omega_{p_1}^2} c^2 q^2 , \tag{2}$$

$$\omega^{\pm} = \pm \frac{\Omega_1 \omega_{p_2}^2 - \Omega_2 \omega_{p_1}^2}{\omega_{p_1}^2 + \omega_{p_2}^2} \pm \frac{\Omega_1^2 \omega_{p_2}^2 + \Omega_2^2 \omega_{p_1}^2}{(\omega_{p_1}^2 + \omega_{p_2}^2)(\Omega_1 \omega_{p_2}^2 - \Omega_2 \omega_{p_1}^2)} c^2 q^2 \tag{3}$$

or, in another notation,

$$\omega^{\mp} = \mp \frac{cHq^2}{4\pi e(n_2 - n_1)}, \tag{4}$$

$$\omega^{\pm} = \pm \frac{eH}{c} \frac{(n_2 - n_1)}{n_1 m_2 + n_2 m_1} \pm \frac{n_1 m_1 + n_2 m_2}{n_1 m_2 + n_2 m_1} \frac{cHq^2}{4\pi e(n_2 - n_1)}. \tag{5}$$

The wave (4) is a helicon [1] that changes its polarization when the sign of the difference $n_2 - n_1$ is changed, while the wave (5) is the new collective excitation. In the local limit ($q \to 0$) this wave can be attributed as collective transverse rotational vibrations of current of uncompensated charge at the "averaged" cyclotron frequency ($\omega^{\pm} < \min\{\Omega_1 \Omega_2\}$). The excitation (5) has the group velocity of the helicon and right (left) circular polarization at $n_1 > n_2$ ($n_2 > n_1$). Its collisionless damping is equal to zero, and the damping due to the collisions is small if $\omega \gg \nu_{\text{eff}}$, where

$$\nu_{\text{eff}} = \frac{\nu_1 n_1 m_1 + \nu_2 n_2 m_2}{n_1 m_1 + n_2 m_2} \tag{6}$$

and ν is the collision frequency. In a metal at $qR^{-1} \ll q_k$ there is collision damping. Therefore the region $q \ll q_k$ is an exhaustion region. To the contrary, in semimetals, for example in Bi, where $q_k \ll R^{-1}$, the excitation (5) exists also at $q_k \lesssim q \ll R^{-1}$. In this region, at $n_2 > n_1$, it has a spectrum

$$\omega = \frac{1}{2}\left\{ \frac{\Omega_1 \omega_{p_2}^2 - \Omega_2 \omega_{p_1}^2 + c^2 q^2 (\Omega_1 - \Omega_2)}{\omega_{p_1}^2 + \omega_{p_2}^2 + c^2 q^2} + \left[\left(\frac{\Omega_1 \omega_{p_2}^2 - \Omega_2 \omega_{p_1}^2 + c^2 q^2 (\Omega_1 - \Omega_2)}{\omega_{p_1}^2 + \omega_{p_2}^2 + c^2 q^2} \right)^2 + \frac{4\Omega_1 \Omega_2 c^2 q^2}{\omega_{p_1}^2 + \omega_{p_2}^2 + c^2 q^2} \right]^{1/2} \right\}. \tag{7}$$

Let us examine the propagation of the excitations at an angle to the direction of the magnetic field: $\vec{q} = (0, q_y, q_z)$. The dispersion equation for this case when $qR \ll 1$ is

$$\left(1 - \frac{4\pi i \omega}{c^2 q^2} \sigma_{xx}\right)\left(1 - \frac{4\pi i \omega}{c^2 q^2 \cos^2\theta} \sigma_{yy}\right) + \left(\frac{4\pi i \omega \sigma_{xy}}{c^2 q^2 \cos\theta}\right)^2 = 0, \tag{8}$$

where σ_{ij} are the components of the conductivity tensor, and its solution in the case when $\omega/\Omega_i \ll 1$ and $q \ll q_k$ are given by the expressions

$$\omega = \frac{cH\cos\theta}{4\pi e|n_2 - n_1|} q^2, \tag{9}$$

$$\omega = \frac{eH}{c}\,\frac{|n_2 - n_1|}{n_1 m_2 + n_2 m_1} + \frac{cH(1+\cos^2\theta)}{8\pi e|n_2 - n_1|}\,q^2. \tag{10}$$

We note that as $\theta \to 0$ the expression (10) does not go over into (5), owing to the limitation $\omega/\Omega_i \ll 1$. When this condition is satisfied, formula (5) coincides with the limiting ($\theta = 0$) value of (10). If $\omega^\pm \ll \omega \ll \Omega_i$ and $q_k \ll q \ll R^{-1}$, then it is easy to verify from (8) that, just as the helicon (9) "goes over" into an Alfven wave [3], the dispersion law of the excitation (10) in the region of the higher frequencies

$$\omega = v_\alpha q(1 + q_k^2/q^2)^{1/2} \tag{11}$$

determines the spectrum of a fast magnetosonic wave ($v_\alpha = H[4\pi(n_1 m_1 + n_2 m_2)]^{-1/2}$ is the Alfven velocity). The helicon (9) has elliptical polarization [1], $E_x/E_y = i\cos\theta$, and for the excitation (10) we have

$$\frac{E_x}{E_y} = -i\,\frac{q_k^2 + q^2(1+\cos^2\theta)/2}{q_k^2 + q^2\cos^2\theta} \tag{12}$$

i.e., it remains circularly polarized when $q^2 \ll q_k^2$.

Let us examine the influence of the excitation (5) on the high-frequency characteristics of a metal, namely the surface impedance and the function $T_\pm(z)(\vec{q}\parallel\vec{H})$ [1], which characterizes the distribution of the waves in the metal. This function and the surface impedance $Z_\pm$ are given by

$$T_\pm(z) = \frac{i\pi}{2k_\pm}\left(1 - i\frac{\gamma}{2}\right)\exp\left[ik_\pm\left(1 + i\frac{\gamma}{2}\right)z\right] \tag{13}$$

if

$$k_\pm^2 = \frac{\omega^2}{v_\alpha^2} \pm \frac{4\pi e\omega(n_2 - n_1)}{cH} > 0 \tag{14}$$

and

$$T_\pm(z) \approx \frac{\pi}{2|k_\pm|}\left(1 + i\frac{\gamma}{2}\right)\exp(-|k_\pm|z).$$

Here $\gamma = (\omega\nu_{eff}/v_\alpha^2 k_\pm^2) \ll 1$ is the ratio of the damping of the wave (5), $\gamma = \nu_{eff}/\omega$, to that of the helicon (4):

$$\gamma = \frac{\nu_1 n_1 m_1 + \nu_2 n_2 m_2}{(n_1 m_2 + n_2 m_1)|\omega^\pm(0)|}, \tag{15}$$

$$Z_\pm = \frac{4\pi\omega}{c^2 k_\pm}\left(1 - i\frac{\gamma}{2}\right),\ k_\pm^2 > 0;\quad Z_\pm = \frac{4\pi\omega}{ic^2 k_\pm}\left(1 + i\frac{\gamma}{2}\right),\ k_\pm^2 < 0.$$

A study of the spatial dispersion of the excitation (5) shows that its spectrum extends to the limit of the Doppler-shifted cyclotron resonance of the electrons (holes) at $n_2 > n_1$ ($n_1 > n_2$) and $\vec{q}\parallel\vec{H}$, and for an arbitrary propagation direction it extends to the limit of the Doppler-shifted cyclotron resonance of the heavy carriers, where the spectrum can have a singularity of the same type as when $\vec{q}\parallel\vec{H}$. In quantizing magnetic fields, the spectrum of the wave (5) near the resonances goes over into the spectrum of the so-called dopplerons. A discussion of these questions is outside the scope of the present article and will be given in a detailed paper.

In conclusion we note that the electromagnetic excitations in an uncompensated Bi plasma at $n_1 \geq n_2$ were investigated experimentally in [4]. It would be of interest to carry out analogous investigations of the new excitation (5).

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CONSTANCY OF TOTAL CROSS SECTIONS AND THE PROCESS $\pi^- p \to \eta n$

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A study of charge-exchange and regeneration processes is of interest because these are the four-particle reactions most sensitive to the model assumptions concerning the interaction mechanism.

In particular, when the question arose of the possible violation of the Pomeranchuk theorem in connection with the obtained Serpukhov-accelerator data on the total hadron-interaction cross sections [1], a number of papers were published [2, 3] devoted to a thorough analysis of these processes with an aim at verifying the Pomerahchuk theorem. The sources of information in these papers are the reactions $\pi^- p \to \pi^0 n$, $K_L A \to K_S A$ (A = matter), and $K^- p \to K^0 n$.

An additional source of information may be the reaction $\pi^- p \to \eta n$, which always accompanies the reaction $\pi^- p \to \pi^0 n$.

In addition, measurement of the reaction $\pi^- p \to \eta n$ can yield valuable information on the connection between this reaction and the charge-exchange reactions $\pi^- p \to \pi^0 n$ and $K^- p \to K^0 n$ within the framework of SU(3) symmetry, or in other words concerning the mechanism of exchange in the t-channel at high energies.

In this paper the amplitude of the process $\pi^- p \to \eta n$ is related with the amplitudes of the reactions $\pi^- p \to \pi^0 n$ and $K^- p \to K^0 n$ within the framework of SU(3) symmetry [4, 5]. The real part of the aforementioned amplitude is reconstructed assuming no contribution is made by the representations 27, 10, and $\overline{10}$ in t-channel, and using dispersion representations. The possible consequences of the constancy of the total cross sections of the elastic processes for the $\pi^- p \to \eta n$ reaction is investigaged. The results are compared with the predictions of the simple Regge-pole model.

Neglect of the contributions of the representations 27, 10, and $\overline{10}$ (in which there are no one-meson states) to the t-channel amplitudes is equivalent to assuming that no account is taken of the contribution made to the amplitude by the multi-meson exchanges contained in these representations. The relative smallness of the discarded terms is suggested by the fact that the amplitudes of the double charge exchange reactions, which are suppressed compared with