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Role of fluctuations in nonlinear dynamics of driven polariton system in semiconductor microcavities

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The escape time from the lower energy state of the bistable nonlinear driven microcavity oscillator have been obtained analitically by means of quasiclassical kinetic equation in the basis of quasienergy states. The dependence of the escape time on the intensity of the external field is in rather good agreement with the results of numerical experiments. Moreover, the numerical dependencies of escape time on damping parameter reveal smooth crossover from exponential to diffusive like behavior.

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Nonlinear phenomena are well known as an infinite source of intriguing physics. Optical nonlinearity usually originates from peculiarities of matter susceptibility and is strongly increased near the resonance frequencies, e.g. exciton resonance in semiconductors. The nonlinear effects for given external pump power are strongly increased in semiconductor microcavities [1] because in these resonant systems the quantum well is located close to the maxima of resonant photon mode. Provided that the frequency of the cavity resonance is close to the frequency of the quantum well exciton and their coupling constant is larger than the dephasing rate, two polariton states (upper and lower) are formed.

In case the external pump frequency exceeds the lower polariton branch (LP) more than the LP-linewidth, the polariton field amplitude can show a bi-stable behavior. With the increase of pump intensity the low density state disappears and the system abruptly jumps to a high density state. Such a jump is accompanied by a strong modification of parametric scattering patterns and pump transmission [2, 3].

An important peculiarity of the cavity polaritons is the possibility of multistability and polarization hysteresis of the coherently driven macro-occupied polariton mode [4]. The multistability arises due to polarization-dependent polariton-polariton interactions and can be revealed in polarization resolved transmission and photoluminescence experiments. With increasing external coherent pumping the driven macroscopically occupied polariton mode shows strong and sudden jumps from one multistable state to another. The existence of different polarization-intensity regimes, corresponding to the same external pumping, is expected to manifest itself in strong spatial and/or temporal fluctuations of the driven polariton wave for driving parameters within multi-stability range.

The presumable strong dependence of transition probabilities on the driving parameters near the stability edge opens a challenging area of controlling these processes. As in any phase transition regime initial fluctuations at the time of transformation can be 'frozen' in the final state spatial distribution function and the latter can be considered as a fingerprint of the former made within a very short time – typically in the picosecond range.

The behavior of the nonlinear oscillator in quasiclassical limit can be successfully analyzed in qausienergy state representation [5 — 7]. The effective hamiltonian in rotating wave approximation for slow varying amplitude can be written as

$$H = -\Delta a^\dagger a + \frac{\alpha}{4}(a^\dagger a)^2 - f(a^\dagger + a), \qquad (1)$$

where a is the operator of polariton amplitude, $\Delta = \hbar(\omega_l - \omega_{LP})$ is the energy detuning between the driving laser field quanta $\hbar\omega_l$ and the polariton resonance energy $\hbar\omega_{LP}$, α is a polariton nonlinearity constant and f is the effective driving field amplitude.

Operators a, $a^\dagger$ correspond to the classical canonical slow variables a, a^* and the eigenvalues of H correspond to the quasienergy E in the classical approach. The hamiltonian (1) results in the following equation of motion for slow varying amplitude

$$i\hbar\frac{da}{dt} = -\Delta a + \alpha a|a|^2 - f. \qquad (2)$$

= 0.075 = 0.01 $Q = d$ (0) *( ) = 0.01

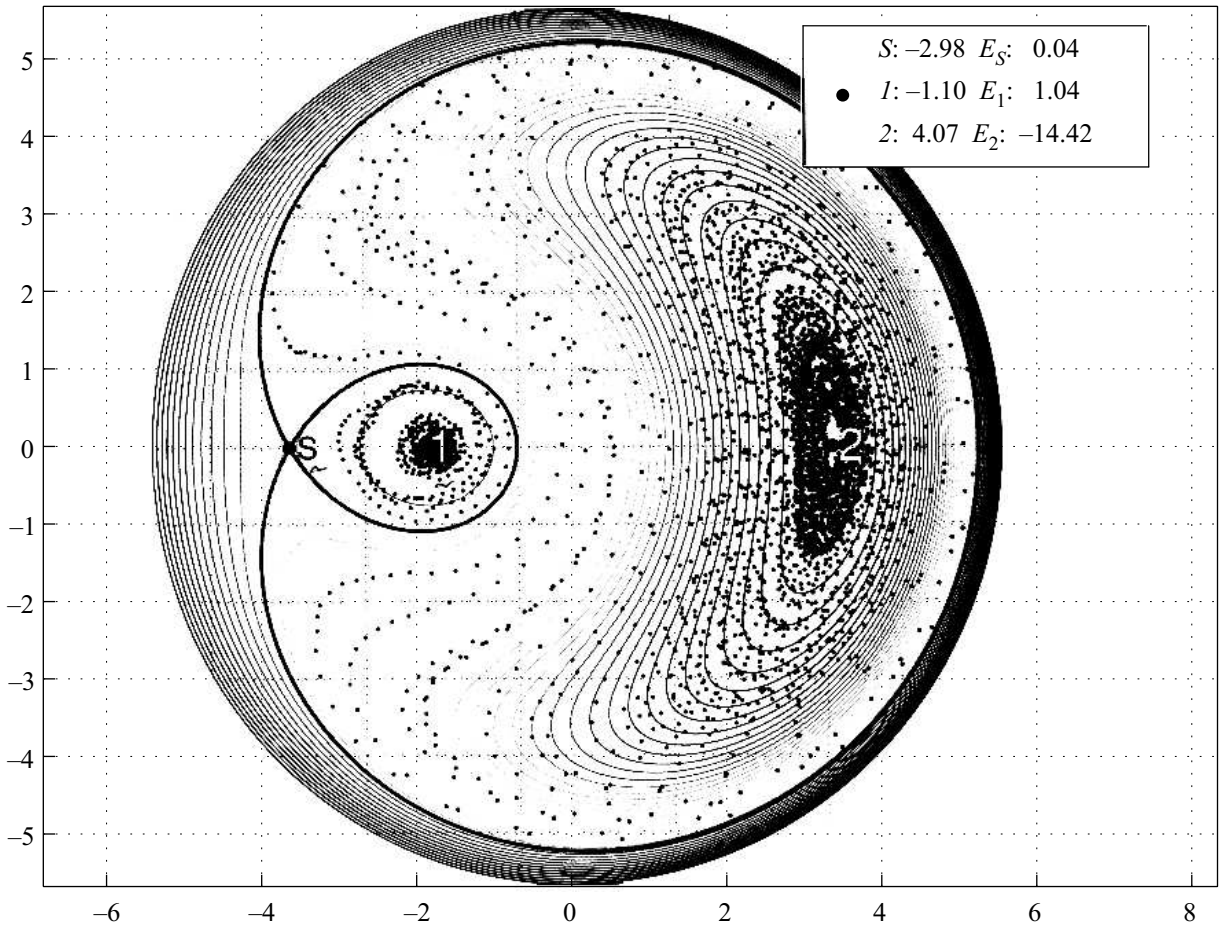

Fig.1. Typical phase portrait of numerical experiment

Transformation to dimensionless variables $a\,\Delta/f \to$ $\to a$ yields

$$i\frac{da}{d\tau} = -a + \beta a|a|^2 - 1, \quad \tau = t\frac{\Delta}{\hbar}. \tag{3}$$

The corresponding dimensionless hamiltonian takes the form

$$H = -a^\dagger a + \frac{\beta}{4}(a^\dagger a)^2 - (a + a^\dagger), \quad \beta = \frac{\alpha f^2}{\Delta^3}. \tag{4}$$

This dimensionless form of the effective hamiltonian depends on a single parameter β that defines the shape of phase trajectories and the probabilities to find the oscillator in different stable quasienergy states.

The nonlinear driven oscillator phase diagram in space ($\mathrm{Re}\,(a)$, $\mathrm{Im}\,(a)$) is depicted in Fig.1. Each phase trajectory corresponds to a particular quasienergy E and $T(E)$ – the period of a particle motion along the corresponding trajectory [8]. The quasienergy is the integral of motion of the nonlinear driven oscillator. It is conserved if dissipation is neglected. Stable states labelled *1* (with smaller oscillation amplitude) and *2* (with lager amplitude) are separated by unstable state laying on phase trajectory called separatrice. E_1, E_2, E_S are the quasienergy values for each trajectory.

Interaction with the thermal bath leads to the appearance of dissipation and fluctuation transitions between the stable states of the nonlinear oscillator. The classical distribution function of different quasienergy states can be obtained by solving the Fokker-Plank equation derived in [7]. Fluctuation transitions between the stable states can lead to the appearance of enhanced probability to find the oscillator in stable state *2* (with lager oscillation amplitude).

To obtain the probability distribution function, we consider the following interaction with the environment

$$\hat{V}_{\mathrm{int}} = \lambda \sum_i (\tilde{b}_i^\dagger + \tilde{b}_i)(a^\dagger + a), \tag{5}$$

where $\tilde{b}_i$ describe the i-th thermal bath oscillator (frequency ω_i) linearly connected with the driven oscillator with the coupling constant λ. The thermal bath oscillators are supposed to be in equilibrium. After transformation to slow variables

$$\tilde{b}_i^\dagger + \tilde{b}_i = b_i^\dagger e^{i\omega_l t} + b_i e^{-i\omega_l t} \tag{6}$$

we can use the resonant approximation and define the interaction hamiltonian as

$$\hat{V}(t) = \hat{f}(t)\hat{a}^\dagger + \hat{f}^\dagger(t)\hat{a}, \tag{7}$$

where

$$\begin{gathered}\hat{f}(t) = \lambda \sum_i \hat{b}_i(t), \\ \hat{f}^\dagger(t) = \lambda \sum_i \hat{b}_i^\dagger(t),\ \hat{b}_i(t) = \hat{b}_i(0) e^{i(\omega_i - \omega_l)t}\end{gathered} \tag{8}$$

and ω_i is the set of thermal bath oscillator frequencies. The quantum kinetic equation for a driven nonlinear oscillator can be obtained by the Keldysh diagram technique [9]. The interaction with the thermal bath is supposed to be weak enough. This means, that in quasiclassical limit

$$E_k - E_{k-1} = \hbar\omega_k \gg \vartheta, \tag{9}$$

where E_k are the quasienergy levels characterized by large integer quantum numbers (k) and ϑ is the width of the quasienergy levels.

In the classical limit the quantum kinetic equation transforms to an one-dimensional Fokker-Plank equation describing the classical diffusion in quasienergy space. So we consider the basis of quasienergy states with eigenvalues E_k determined by hamiltonian (4). Using this basis, the ensemble of the noninteracting driven nonlinear oscillators can be described by the effective hamiltonian

$$H_0 = \sum_k E_k c_k^\dagger c_k, \tag{10}$$

where c_k ($c_k^\dagger$) is the operator which annihilates (creates) the oscillator in quasienergy state E_k and

$$\langle c_k^\dagger c_k \rangle = n(E_k, t) \tag{11}$$

is the probability distribution to find the oscillator in quasienergy state E_k. We also define V_{int} arising from the interaction with the thermal bath in quasi-energy representation

$$\hat{V}_{\text{int}} = \sum_{k,k'} a_{k,k'} \hat{f}^\dagger(t) c_k^\dagger c_{k'} + a_{k,k'}^\dagger \hat{f}(t) c_{k'}^\dagger c_k, \tag{12}$$

with f as the random force acting in slow variables space.

By means of Keldysh diagram technique we obtain in selfconsistent quasiclassical limit the kinetic equation for $G_{k,k}^<$:

$$i\hbar \frac{\partial}{\partial t} G_{k,k}^< = \Sigma_{k,k}^<(\omega) G_{k,k}^>(\omega) - \Sigma_{k,k}^>(\omega) G_{k,k}^<(\omega). \tag{13}$$

In diagonal approximation $G_{k,k}^<$ satisfies the equation (13) with

$$\Sigma_{k,k}^{<(>)}(\omega) = \int d\omega' D_{k,k'}^{<(>)}(\omega') G_{k',k'}^{<(>)}(\omega - \omega'), \tag{14}$$

$$D_{k,k'}^<(\omega) = \lambda^2 \nu \left\{ |a_{k,k'}|^2 (N_{\omega_l - \omega} + 1) + |a_{k',k}|^2 N_{\omega_l + \omega} \right\}, \tag{15}$$

$$D_{k,k'}^>(\omega) = \lambda^2 \nu \left\{ |a_{k,k'}|^2 N_{\omega_l - \omega} + |a_{k',k}|^2 (N_{\omega_l + \omega} + 1) \right\}, \tag{16}$$

where ν is the density of states of the thermal bath oscillators and N_ω is the filling number of ω-frequency thermal bath oscillator. The Keldysh Green function $G_{k,k}^<$ is defined in usual way:

$$G_{k,k}^<(\omega) = -2\pi i n_k(\omega) \mathrm{Im} G_{k,k}^R(\omega). \tag{17}$$

If the width of quasienergy levels is much smaller than the difference between quasienergies, $G_{k,k}^<(\omega)$ simplifies to

$$G_{k,k}^<(\omega) = -2\pi i n_k(\omega) \delta(\omega - E_k). \tag{18}$$

The same expressions with substitution n_k by $(1 - n_k)$ are valid for $G_{k,k}^>(\omega)$. If $E_{k1(2)} - E_k \ll E_{k1(2)}$, where $E_{k1(2)}$ corresponds in classical limit to E_1 and E_2, a rather large number of quasienergy levels close to $E_{k1(2)}$ are exited and the probabilities n_k are small. Thus, in quasiclassical limit we retain only linear n_k-terms in the kinetic equation. We obtain from (13)–(18) the kinetic equations for n_k – the probability to find the system in quasienergy state E_k – by integration over ω neglecting the broadening of quasienergy levels.

$$\frac{\partial n_k}{\partial t} = \frac{2\pi}{\hbar} \sum_{k'} D_{k,k'}^< n_{k'} - D_{k,k'}^> n_k. \tag{19}$$

In quasiclassical approximation (19) can be transformed to a Fokker-Plank equation in each region of phase-space 1 and 2 with quasienergy depending coefficients. The matrix elements $a_{k,k'}$ are quickly changing away from the diagonal $k = k'$ with increasing of $q = k - k'$ as compared to their slow variations with k along the diagonal in the same region of phase space i.e. $|a_{k,k+q}|^2 \sim |a_{k-q,k}|^2$ for $q/k \ll 1$.

The right hand side of the equation (19) can be expanded in series up to q^2 terms [10] and the quasi-classical approximation for matrix elements [11] can be used:

$$\sum_q |a_{k,k+q}|^2 q = \tag{20}$$

$$= (2\pi\omega(E_k))^2 \sum_q q \left| \int_0^{T(E_k)} a(t) e^{i\omega(E_k)qt} dt \right|^2 =$$

$$= \frac{1}{2i\omega(E_k)T(E_k)} \int_0^{T(E_k)} (a(t)\dot{a}^*(t) - a^*(t)\dot{a}(t))dt =$$

$$= \frac{1}{2i} \oint_{C(E_k)} ada^* - a^* da,$$

$$\sum_q |a_{k,k+q}|^2 q^2 =$$

$$= \frac{1}{2i\omega^2(E_k)T(E_k)} \oint_{C(E_k)} \frac{\partial H}{\partial a} da - \frac{\partial H}{\partial a^*} da^*. \tag{21}$$

In these expressions $T(E_k)$ and $\omega(E_k)$ are the period and frequency of motion along the phase trajectory C with quasienergy E_k. In addition, we should take into account that in quasi-classical limit $E_k - E_{k-q} = \hbar\omega(E_k)q$, i.e. $\partial E_k/\partial k = \hbar\omega(E_k)$. If $E_k - E_{k'} < \hbar\omega_l$, we can consider that $N_{\omega_l \pm \omega} \sim N_{\omega_l}$. Thus, the kinetic equation (19) reduces to an one dimensional Fokker-Plank equation in each region of phase-space for $n_i = n_i(E, \tau)$:

$$\frac{\partial}{\partial \tau} n_i = 2\pi \frac{\partial}{\partial E} \left(\vartheta K(E) n_i + Q D(E) \frac{\partial}{\partial E} n_i \right), \tag{22}$$

where $i = 1, 2$.

In (22) K and D are determined by integrals along the phase trajectories with particular quasienergy value E - closed curve $C(E)$ in dimensionless variables:

$$K(E) = \frac{1}{2iT(E)} \oint_{C(E)} ada^* - a^* da,$$
$$D(E) = \frac{1}{2iT(E)} \oint_{C(E)} \frac{\partial H}{\partial a} da - \frac{\partial H}{\partial a^*} da^*. \tag{23}$$

with

$$Q = \frac{1}{2}(2N_{\omega_l} + 1)\vartheta\eta, \quad \vartheta = \frac{\lambda^2 \nu}{\Delta} \tag{24}$$

as the random force intensity. The coefficient η arises from transition to dimensionless variables (a, a^*):

$$\eta = \Delta^2 / f^2. \tag{25}$$

The stationary solution of (22) for the probability to find the system in quasienergy state E in the vicinity of stable points can be written as

$$n_{1(2)}(E) = A_{1(2)} \exp\left[-\frac{\vartheta}{Q} \int_{E_{1(2)}}^{E} dE' \frac{K_{1(2)}(E')}{D_{1(2)}(E')} \right], \tag{26}$$

and the continuity of the distribution function at the saddle point quasienergy value yields

$$n_1(E_S) = n_2(E_S). \tag{27}$$

The relative population of the two stable states of the driven nonlinear oscillator can be easily obtained from (26) and (27)

$$\frac{n_1(E_1)}{n_2(E_2)} = \frac{A_1}{A_2} = \frac{\exp\left[-\frac{\vartheta}{Q} \int_{E_2}^{E_S} dE' \frac{K_2(E')}{D_2(E')} \right]}{\exp\left[-\frac{\vartheta}{Q} \int_{E_1}^{E_S} dE' \frac{K_1(E')}{D_1(E')} \right]}. \tag{28}$$

To calculate the escape rates from the stable state 1 and 2 it is helpful to transform the Fokker-Plank equation into a Schrцdinger equation. Using the substitution

$$n_i(E, t) = \frac{1}{\sqrt{D(E)}} \exp\left[-\frac{\vartheta}{Q} \int^{E} dE' \frac{K_i(E')}{D_i(E')} \right] F(E, t), \tag{29}$$

we obtain the Schrüdinger equation for F with the effective potential determined by kinetic coefficients K and D.

$$\frac{Q}{\vartheta} \frac{\partial F}{\partial t\vartheta} = -\frac{Q^2}{\vartheta^2} D(E) \frac{\partial^2 F}{\partial^2 E} + V(E) F, \tag{30}$$

$$V(E) = \frac{K^2}{4D} + \frac{2K'D - D'K}{2D} \frac{Q}{\vartheta}. \tag{31}$$

The escape rate from the each stable state corresponds now to the tunnelling transition amplitude from the stable point $E_{1(2)}$ to E_S. The escape time from state 1 can be evaluated as

$$\tau_1 \sim (2\pi\vartheta)^{-1} \times$$
$$\times \left| \exp \frac{\vartheta}{2Q} \int_{E_1}^{E_S} dE' \sqrt{\frac{K^2}{D^2} + \frac{2K'D - D'K}{D^2} \frac{Q}{\vartheta}} \right|^2. \tag{32}$$

The classical analog of equation (32) was derived in [7] staring from the classical stochastic equations of motion for slow variables a and a^+ and averaging over $T(E)$ the corresponding two dimensional Fokker-Plank equation.

In the general case, the random noise can arise not only from the interaction with the thermal bath, but

also because of inevitable fluctuations of the pump. As a result, the damping coefficient can be negligibly small while the noise intensity remains finite. The opposite situation with a decay time larger than the noise intensity is also possible for the studied polariton system due to the short polariton lifetime. Therefore we consider here noise and decay time as two independent parameters.

In the case of zero damping, the oscillator probability distribution is determined by a diffusion equation with quasienergy dependent diffusion coefficient. The time dependent distribution function can be easily obtained with exponential accuracy

$$n(E,t) \sim \exp\left\{\frac{1}{4Qt}\left[\int_{E_1}^{E}\frac{dE}{\sqrt{D(E)}}\right]^2\right\} \qquad (33)$$

and the mean escape time from the stable state with the quasienergy E_1 can be evaluated as

$$\tau \sim \frac{1}{4Q}\left[\int_{E_1}^{E_S}\frac{dE}{\sqrt{D(E)}}\right]^2. \qquad (34)$$

We compare the analytical expressions for the mean escape time τ_1 with numerical solutions of the following equation of motion for the slow varying amplitude in the presence of dissipation ϑ and random noise ξ:

$$i\frac{da}{d\tau} = -a - i\vartheta a + \beta a|a|^2 - 1 + \xi(\tau), \qquad (35)$$

$$\langle\xi^*(0)\xi(\tau)\rangle = Q\delta(\tau), \quad \langle\xi(0)\xi(\tau)\rangle = 0. \qquad (36)$$

The averaged inverse escape time from the low amplitude quasi stable state $\tau_1 = \langle\tau_j^{-1}\rangle^{-1}$ has been calculated over 100 numerical solutions for each set of parameters β, ϑ and Q. The explicit relations between the latter dimensionless parameters and noise spectral density σ^2, pump amplitude (f), pump detuning (Δ), oscillator dephasing (γ) and nonlinearity constant (α) as well as escape time t_1 and dimensionless time τ_1 are given below.

$$\beta = \frac{\alpha f^2}{\Delta^3}, \quad \vartheta = \frac{\gamma}{\Delta}, \quad Q = \frac{\sigma^2}{f^2}\frac{\Delta}{\hbar}, \quad t_1 = \tau_1\frac{\hbar}{\Delta}. \qquad (37)$$

Figs.2 and 3 show the dependence of τ_1 on the parameter β and on dumping coefficient ϑ for different noise intensities Q. The qualitative features of numerically obtained escape time dependencies on parameter β are reproduced by analytical expression (33), where the preexponential factor is not taken into account. The analytically obtained values of the exponent are of the order of $10^2 - 10^3$ (Fig.2), and are large compared to the ratio between the analytical and numerical values of

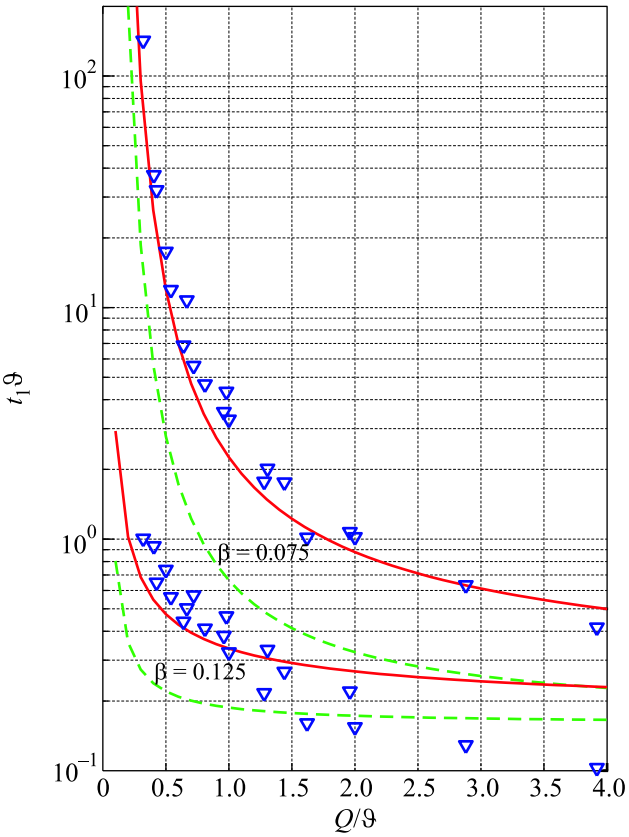

Fig.2. Product of dimensionless escape time and dephasing ($\tau_1\vartheta$) vs dimensionless noise to dephasing ratio for two values of parameter $\beta = 0.075$ and 0.125. Solid lines are calculated according to Eq.(32) while the dashed lines are calculated neglecting the second term under the square root in the same equation. Triangles show the same dependence obtained from numerical experiments for $\tau_1 = \langle\tau_j^{-1}\rangle^{-1}$ by averaging of inverse escape time over 100 random noise realizations

escape time τ_1. Thus, it is reasonable to ascribe this ratio to preexponential factor which is determined numerically and is absent in analytical expressions (32) and (34).

Let's illustrate these results for pump detuning from resonance $\Delta = 1$ meV and the resonance width $\gamma = 0.2$ meV. The plot in Fig.2 for $\beta = 0.125$ corresponds to pump intensity 15% less the upper boundary of bistability range ($\beta_c = 4/27$) and gives $\tau_1\vartheta \approx 1$ for $Q/\vartheta \approx 0.2$. Using (37) we get $\tau_1\vartheta = t_1\Delta/\hbar\cdot\gamma/\Delta = t_1\gamma/\hbar$ and $Q/\vartheta = \sigma^2/f^2\cdot\Delta/\hbar\cdot\Delta/\gamma = (\sigma^2\gamma/\hbar)/f^2\cdot(\Delta/\gamma)^2$. This means the escape time $t_1 \approx \hbar/0.2\,\text{meV} \approx 3\,\text{ps}$ for the noise intensity within the resonance linewidth to pump intensity ratio $(\sigma^2\gamma/\hbar)/f^2 \approx 0.01$.

Further, we have investigated numerically the zero damping diffusive regime of fluctuation transitions of nonlinear driven oscillator for different noise intensities.

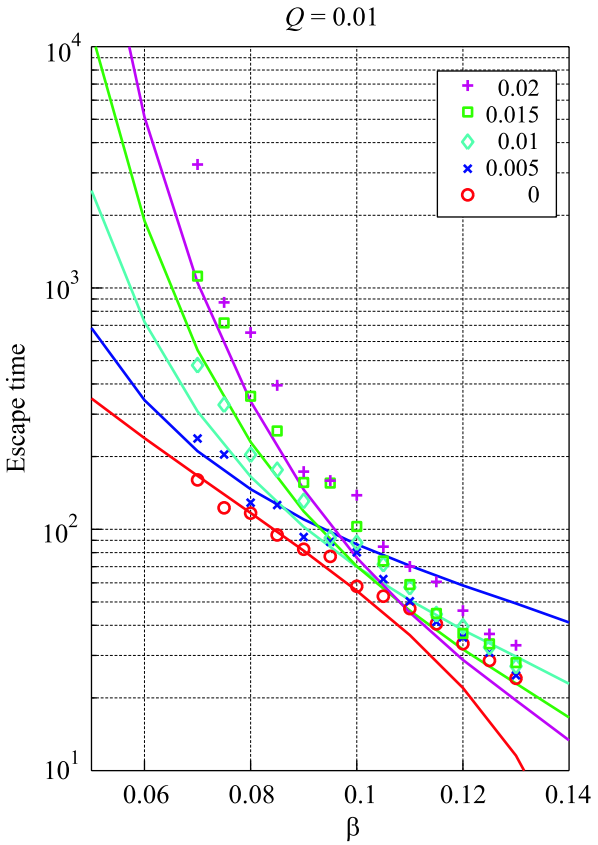

Fig.3. Escape time (τ_1) vs dimensionless nonlinear parameter (β) for dephasing constants ϑ shown in the legend. Solid lines are calculated according to Eq.(32) where as markers show $\tau_1 = \langle \tau_j^{-1} \rangle^{-1}$ obtained from numerical experiments

The values of the escape time given by (34) coincides with the numerical results shown in Fig.3. The behavior of τ_1 in the range of small damping

$$0 < \vartheta < \frac{Q}{\left[\int_{E_1}^{E_S} dE \frac{K(E)}{D(E)}\right]}, \qquad (38)$$

where the exponential accuracy is insufficient, has been analyzed numerically. The calculated dependencies of escape time τ_1 on the damping constant reveal a smooth crossover from the behavior given by (32) to diffusive like behavior given by (34), see Fig.3.

In conclusion we want to point out that fluctuation induced transitions between two stable states of driven nonlinear cavity polariton oscillator can be analyzed by means of one-dimensional kinetic equation in quasi-energy representation. The analytically obtained escape times from the lower energy stable state are in a good agreement with the results of numerical experiments for various system parameters.

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